AP Government GATE Online Classes

Heat Transfer

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RADIATION BETWEEN SURFACES

Radiation between Surfaces:

The radiation Heat Transfer between different types of surfaces both in non-participating & participating media, the following assumptions be made:

1. All surfaces have uniform property over their whole extent.
2. Each surface is considered to be either gray or black.
3. The absorptivity of surface is independent of the 'Temp' of the source of the incident radiation & equal its emissivity.
4. Radiation & reflection processes are diffused.

Radiation between Surfaces:

- For any two given surfaces, the orientation between them affects the fraction of radiation energy leaving one surface and that strikes the other.

- To take into account this, the concept of view factor/ shape factor/ configuration factor is introduced.

- The physical significance of the view factor between two surfaces is that it represents the fraction of the radiative energy leaving one surface that strikes the other surface directly.

Solid Angle (or) Angle of vision

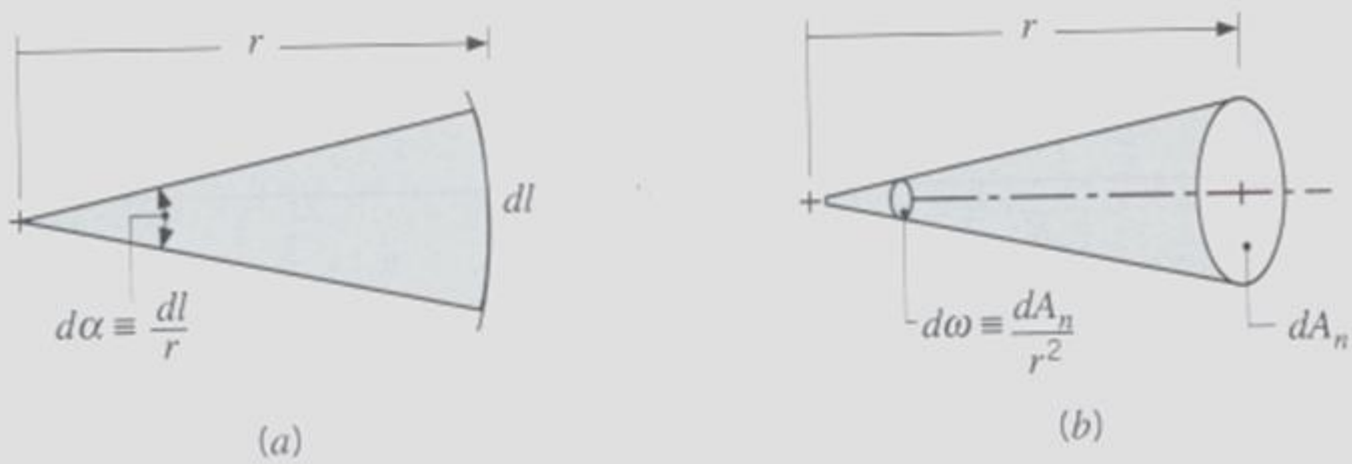
- Qualitatively the interception of radiation from an area element of a surface by another surface of finite size can be visualized in terms of the angle of vision, which is the solid angle subtended by the finite surface at the radiating element.

- The solid angle subtended by a hemisphere is 2Π steradians. This is the maximum angle of vision that can be subtended at any area element by a plane surface insight of the element.

- If the angle of vision is $< 2\Pi$ steradians, only a fraction of the radiation from the area element will be intercepted by the receiving area & the remainder will pass on, to be absorbed by other surfaces in sight of the remaining solid angle.

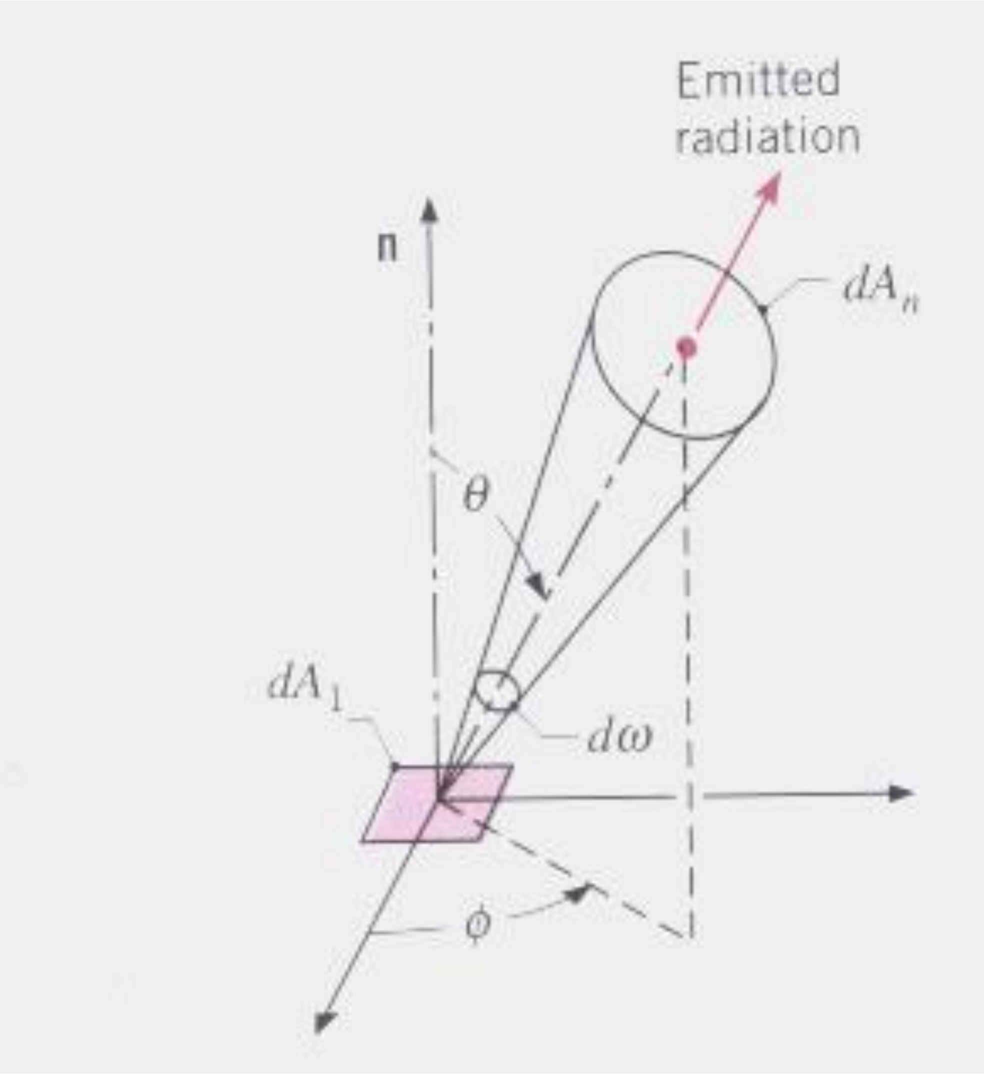
Solid Angle (or) Angle of vision

- differential solid angle $d\omega = dA_n/r^2 = dA_1 \cos\theta/r^2$
- A solid angle is for a sphere what an angle is for a circle
- Units: steradians (sr); For a sphere $\omega = 4\pi$ sr



Definition of (*a*) plane and (*b*) solid angles.

Solid Angle, cont.



Plane Angle and Solid Angle

$$\omega = \frac{A_n}{r^2} = \frac{A\cos\theta}{r^2}$$

A_n: projection of the incident surface normal to the line of projection

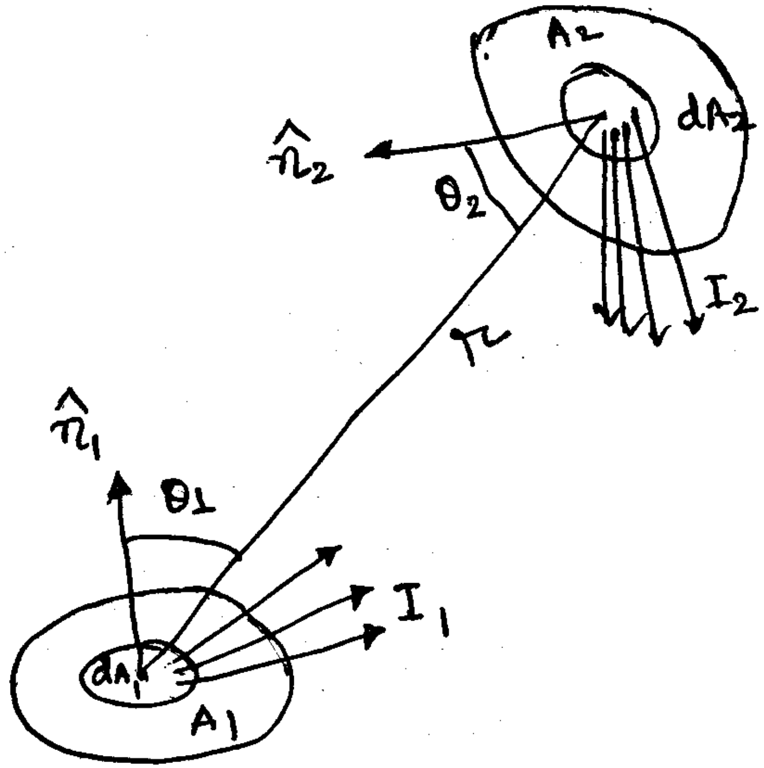
θ: angle between the normal to the incident surface and the line of propagation.

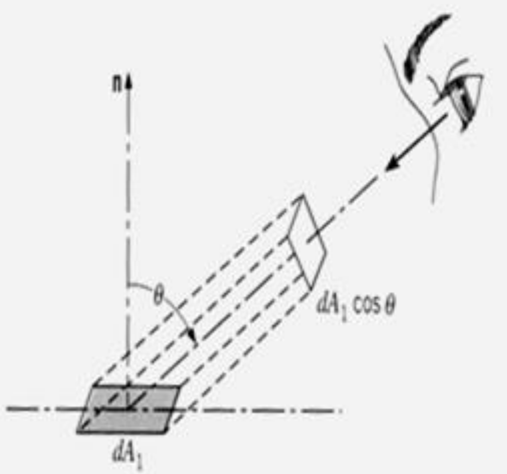
r: length of the line of propagation between the radiating and the incident surfaces

View factor between two elemental surfaces

- Consider two elemental areas dA_1 and dA_2 on body 1 and 2 respectively.
- Let $d\omega_{12}$ be the solid angle under which an observer at dA_1 sees the surface element dA_2 and
- I_1 be the intensity of radiation leaving the surface element diffusely in all directions in hemispherical space.

View Factor Figure





The projection of dA_1 normal to the direction of radiation.

View factor

Therefore, the rate of radiative energy dQ_1 leaving dA_1 and strikes dA_2 is

$$dQ_{12} = dA_1 I_1 \cos\theta_1 d\omega_{12}$$

where solid angle $d\omega_{12}$ is given by

$$d\omega_{12} = \frac{dA_2 \cos\theta_2}{r^2}$$

View factor

Combining the equations, we get

$$dQ_{12} = dA_1 I_1 \frac{\cos\theta_1 \cos\theta_2 dA_2}{r^2}$$

Now, the intensity of normal radiation is given by

$$I_1 = \frac{E_b}{\pi} = \frac{\sigma_b T_1^4}{\pi}$$

Shape Factor

$$\therefore dQ_{12} = \frac{\sigma_b T_1^4}{\pi} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2}$$

Now, we define shape factor, F_{12} as

$$F12 = \frac{\text{direct radiation from surface1 incident on surface2}}{\text{total radiation from emitting surface1}} = \frac{Q_{12}}{\sigma_b A_1 T_1^4}$$

Shape factor

$$= \frac{1}{\sigma_b A_1 T_1^4} \frac{\sigma_b T_1^4}{\pi} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2}$$

$$= \frac{1}{A_1 \pi} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2}$$

Radiant Heat Transfer Between Two Bodies

The amount of radiant energy leaving A_1 and striking A_2 may be written as

$$Q_{12} = A_1 F_{12} \sigma_b T_1^4$$

Similarly, the energy leaving A_2 and arriving A_1 is

$$Q_{21} = A_2 F_{21} \sigma_b T_2^4$$

Radiant Heat Transfer Between Two Bodies

So, net energy exchange from A_1 to A_2 is

$$\left(Q_{12}\right)_{net} = A_1 F_{12} \sigma_b T_1^4 - A_2 F_{21} \sigma_b T_2^4$$

When the surfaces are maintained at the same temperatures, $T_1 = T_2$, there cannot be any heat exchange between them.

$$\because 0 = A_1 F_{12} \sigma_b T_1^4 - A_2 F_{21} \sigma_b T_2^4$$

$$A_1 F_{12} = A_2 F_{21}$$

Reciprocity theorem

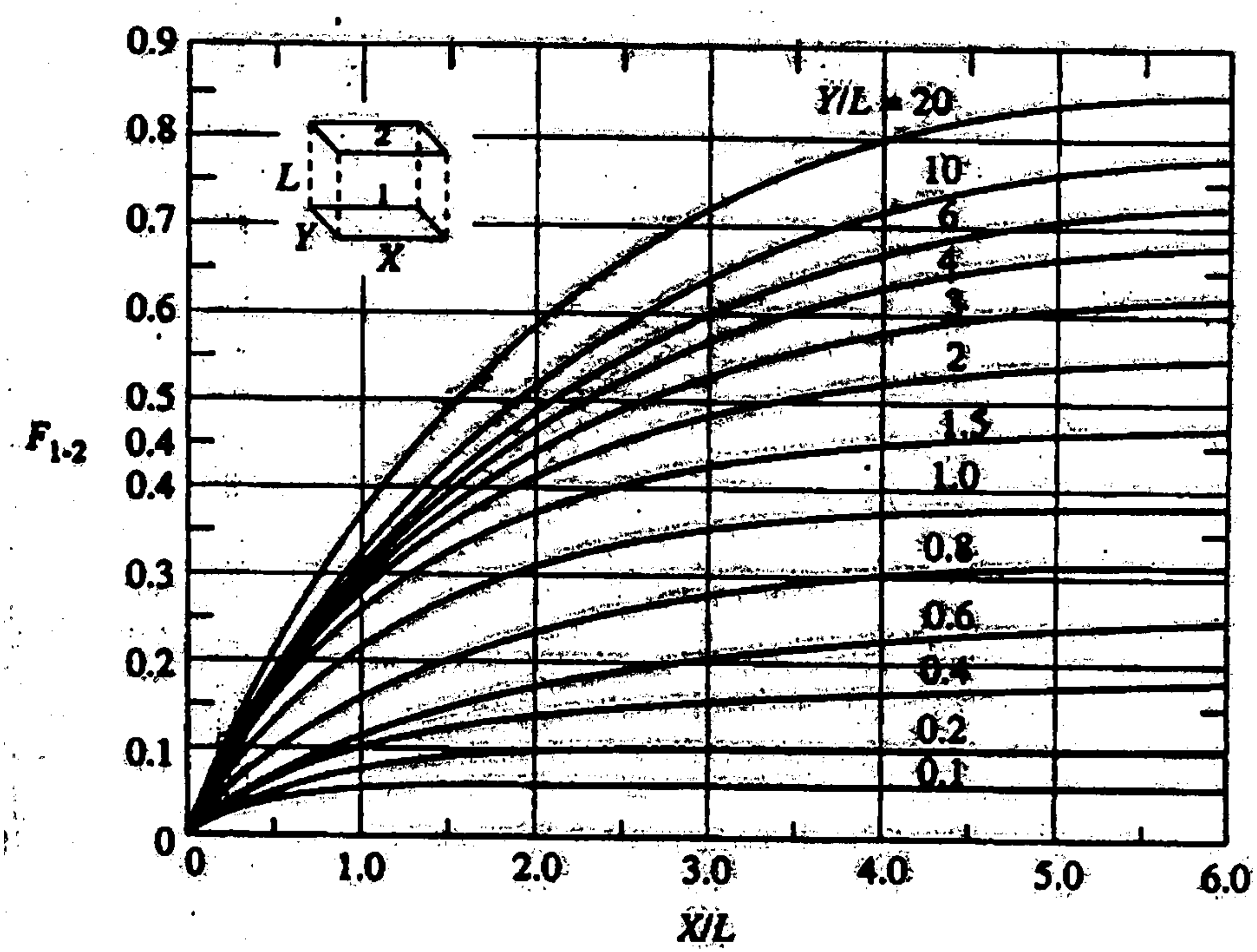
Net Heat transfer

$$\left(Q_{12}\right)_{net} = A_1 F_{12} \sigma_b \left(T_1^4 - T_2^4\right) = A_2 F_{21} \sigma_b \left(T_1^4 - T_2^4\right)$$

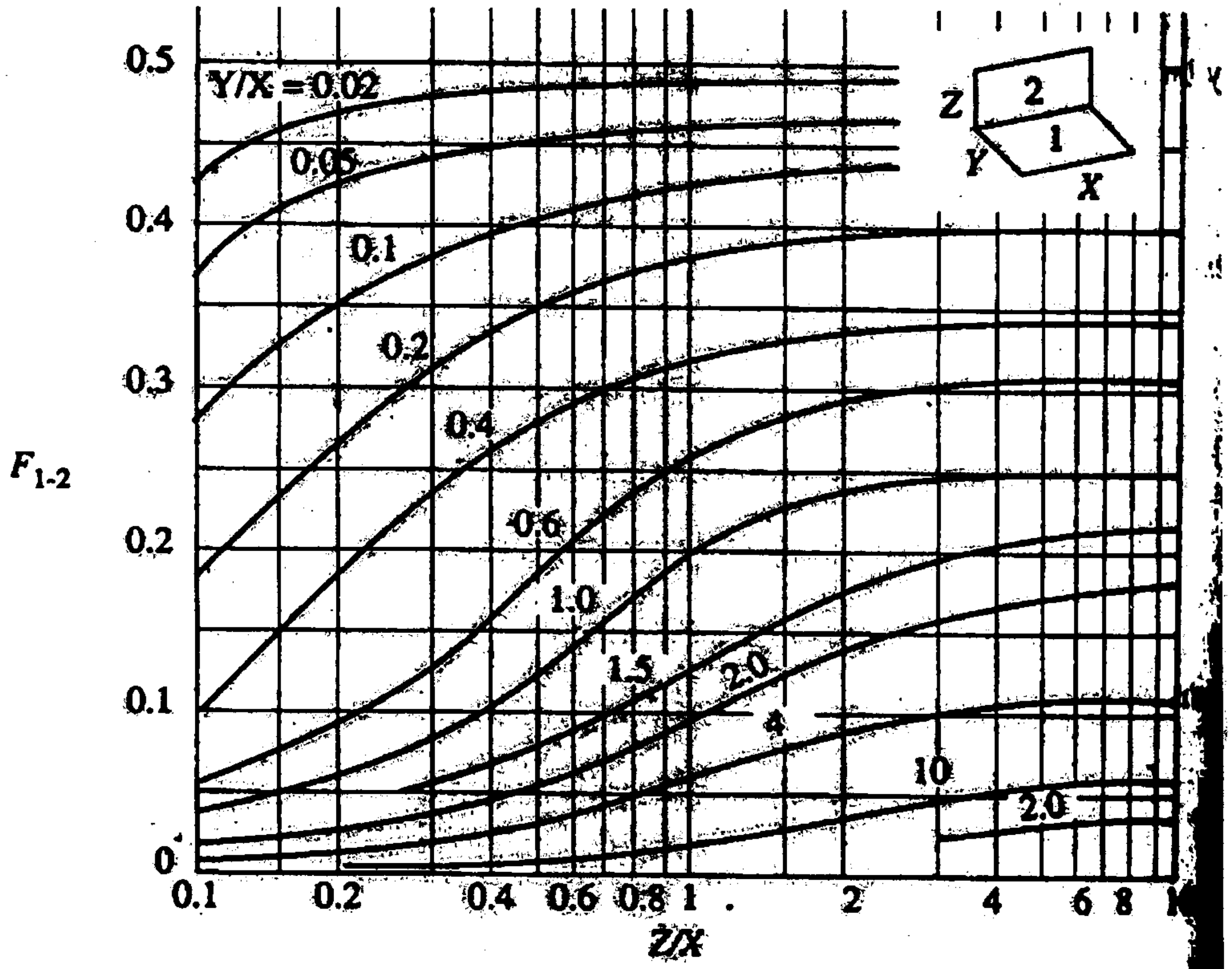
The evaluation of the integral for the determination of shape factor for complex geometries is rather complex and cumbersome.

Results have been obtained and presented in graphical form for the geometries normally encountered in engineering practice.

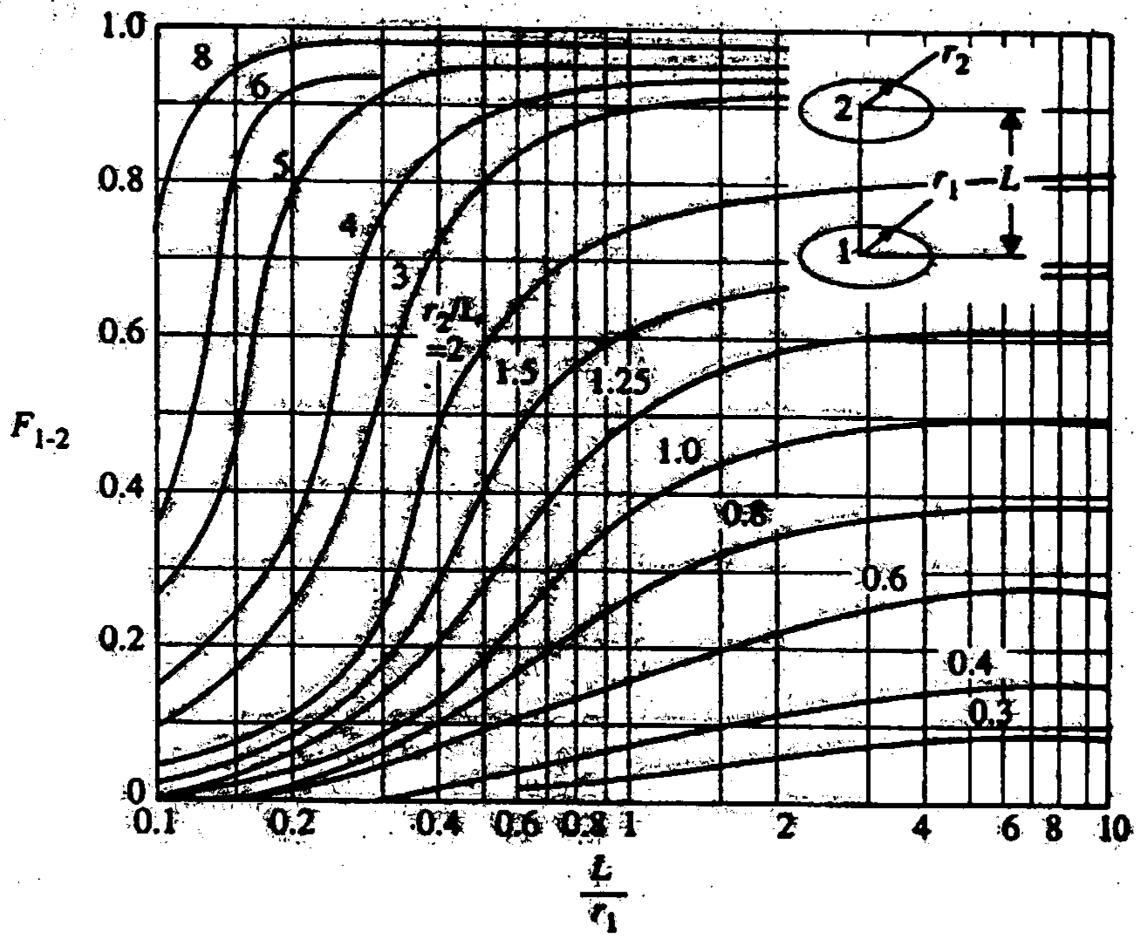
SHAPE FACTOR FOR ALLIGNED PARALLEL PLATES



SHAPE FACTOR FOR PERPENDICULAR RECTANGLES WITH COMMON BASE



SHAPE FACTOR FOR COAXIAL PARALLEL PLATES



SHAPE FACTOR ALGEBRA

- The shape factors for complex geometries can be derived in terms of known shape factors for other geometries.
- For that the complex shape is divided into sections for which the shape factor is either known or can be readily evaluated.
- The unknown configuration factor is worked out by adding and subtracting known factors of related geometries.
- The method is based on the definition of shape factor, the reciprocity principle and the energy conservation law.

Some Features of Shape Factor

- The value of the shape factor depends only on the geometry and orientation of surfaces with respect to each other. Once the shape factor between two surfaces is known, it can be used for the calculating heat exchange between two surfaces at any temperature.
- All the radiation coming out from a convex surface 1 is intercepted by the enclosing surface 2. The shape factor of convex surface with respect to the enclosure (F_{12}) is unity.
- The radiant energy emitted by a concave surface is intercepted by another part of the same surface. A concave surface has a shape factor with respect to itself and it is denoted by F_{11}. For a convex and flat surface , $F_{11} = 0$.

Features of Shape Factor

If one of the two surfaces (say A_i) is divided into sub-areas A_{i1}, A_{i2}, …., A_{in}, then

$$A_i F_{ij} = \sum A_{in} F_{inj}$$

Shape Factor Algebra

- Any radiating surface will have finite area and therefore will be enclosed by many surfaces.
- The total radiation being emitted by the radiating surface will be received and absorbed by each of the confining surfaces.
- Since shape factor is the fraction of total radiation leaving the radiating surface and falling upon a particular receiving surface.

$$\sum_{j=1}^{n} F_{ij} = 1 \quad , \; i = 1, 2, \ldots\ldots, n$$

Shape Factor Algebra

If the interior surface of a complete enclosed space has been subdivided in n parts having finite area A_1, A_2, …. A_n, then

$$F_{11} + F_{12} + F_{13} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + F_{1n} = 1$$

$$F_{21} + F_{22} + F_{23} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + F_{2n} = 1$$

$$F_{31} + F_{32} + F_{33} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + F_{3n} = 1$$

$$F_{n1} + F_{n2} + F_{n3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + F_{nn} = 1$$

HEAT EXCHANGE BETWEEN NON-BLACK BODIES

- The black bodies absorb the entire incident radiation and this aspect makes the calculation procedure of heat exchange between black bodies rather simple.
- One has to only determine the shape factor.
- However, the real surfaces do not absorb the whole of the incident radiation: a part is reflected back to the radiating surface.
- Also the absorptivity and emissivity are not uniform in all directions and for all wavelengths.

(1).Infinite parallel planes

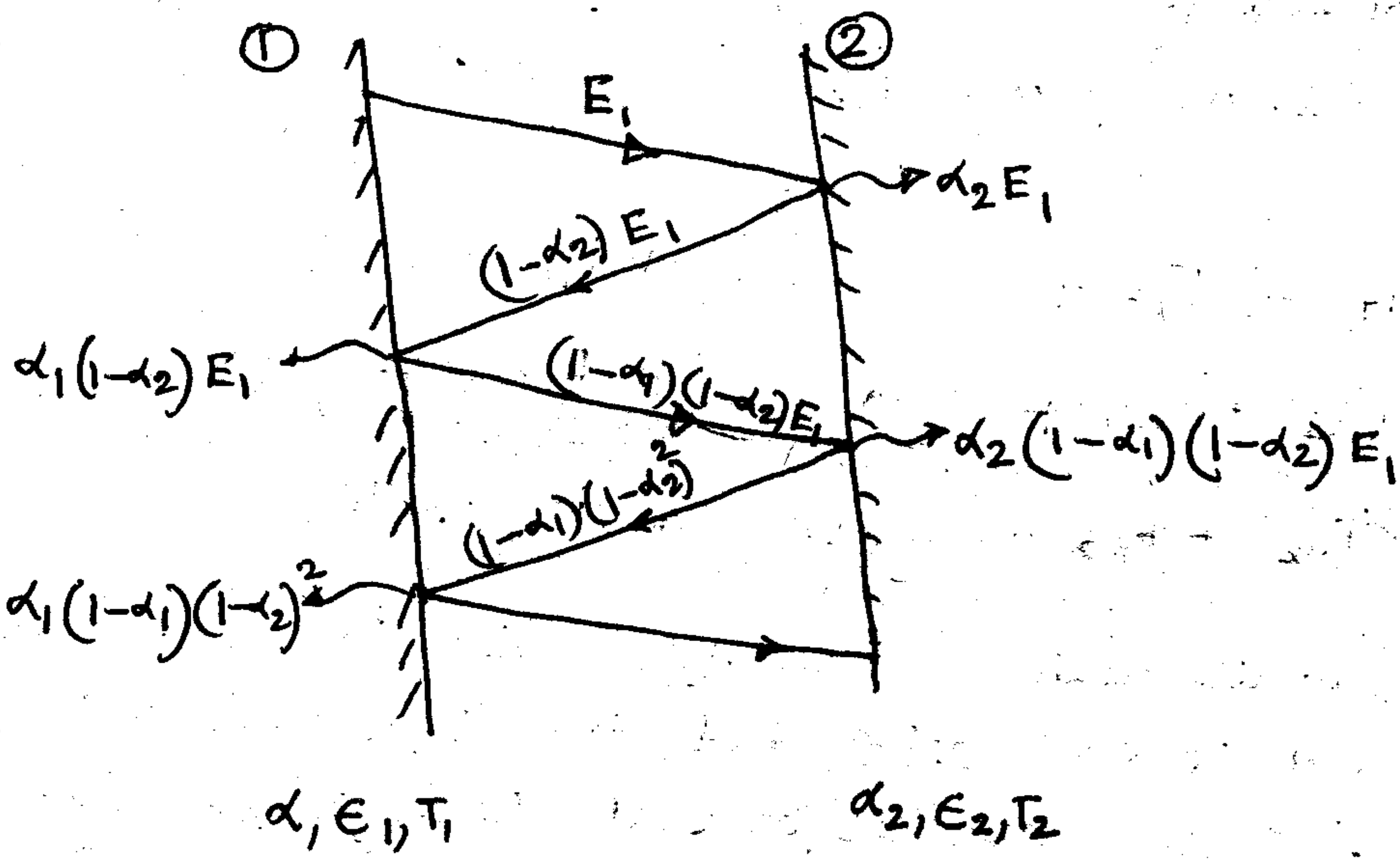
Assumptions

(i) Surfaces are arranged at small distance from each other and are of equal areas so that practically all radiation emitted by one surface falls on the other. The shape factor of either surface is therefore unity.

(ii) Surfaces are diffuse and uniform in temperature, and that the reflected and emissive properties are constant over all the surface.

(iii) The surfaces are separated by a non-absorbing medium as air.

Infinite parallel planes



Heat Transfer between Infinite parallel planes

The amount of radiant energy which left surface 1 per unit time is

$$Q_1 = E_1 - \left[\alpha_1(1-\alpha_2)E_1 + \alpha_1(1-\alpha_1)(1-\alpha_2)^2 E_1 + \alpha_1(1-\alpha_1)^2(1-\alpha_2)^3 E_1 + --------\right]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1\left[1 + (1-\alpha_1)(1-\alpha_2) + (1-\alpha_1)^2(1-\alpha_2)^2 + --------\right]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1\left[1 + p + p^2 + --------\right]$$ where $p = (1-\alpha_1)(1-\alpha_2)$

$$1 + p + p^2 + ------------ \text{upto } \alpha = \frac{1}{1-p}$$

p is less than unity

Calculations

$$Q_1 = E_1 - \frac{\alpha_1(1-\alpha_2)E_1}{1-p}$$

$$= E_1\left[1 - \frac{\alpha_1(1-\alpha_2)}{1-(1-\alpha_1)(1-\alpha_2)}\right]$$

$$= E_1\left[\frac{\alpha_2}{\alpha_1 + \alpha_2 - \alpha_1\alpha_2}\right]$$

$$= E_1\left[\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2}\right] \quad \text{as } \alpha = \varepsilon \text{ from Kirchoff's law}$$

Surface 2

Similarly, the amount of heat which leaves surface 2

$$Q_2 = E_2\left[\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}\right]$$

Therefore, the net heat flow from surface 1 to surface 2 per unit time is given by

$$Q_{12} = Q_1 - Q_2 = E_1\left[\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}\right] - E_2\left[\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}\right]$$

$$= \frac{E_1\alpha_2 - E_2\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

Black Surface

Now, for the black surfaces,

$$E_1 = \alpha_1 \sigma_b T_1^{\,4} \qquad E_2 = \alpha_2 \sigma_b T_2^{\,4}$$

$$\therefore Q_{12} = \frac{\varepsilon_1 \sigma_b T_1^{\,4} \varepsilon_2 - \varepsilon_2 \sigma_b T_2^{\,4} \varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

$$= \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \sigma_b \left(T_1^{\,4} - T_2^{\,4} \right)$$

$$= f_{12} \sigma_b \left(T_1^{\,4} - T_2^{\,4} \right)$$

where f_{12} is called the interchange factor for the radiation from surface 1 to surface 2 and is given by.

Interchange Factor

$$f_{12} = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

(3).Small Gray Bodies

- Small bodies signify that their sizes are very small compared to the distance between them.
- The radiant energy emitted by surface 1 would be partly absorbed by surface 2 and the unabsorbed reflected portion will be lost in space.
- It will not be reflected back to surface 1 because of its small size and large distance between the two surfaces.

Calculations for Small Gray Bodies

Energy emitted by body 1 = $A_1 \varepsilon_1 \sigma_b T_1^4$

Energy incident on by body 2 = $F_{12} A_1 \varepsilon_1 \sigma_b T_1^4$

Energy absorbed by surface 2 = $\alpha_2 F_{12} A_1 \varepsilon_1 \sigma_b T_1^4$

$$\therefore Q_1 = \varepsilon_1 \varepsilon_2 A_1 F_{12} \sigma_b T_1^4$$

putting $\alpha_2 = \varepsilon_2$

Calculations for Small Gray Bodies (2)

Similarly, energy transfer from surface 2 to 1 is

$$\therefore Q_2 = \varepsilon_1 \varepsilon_2 A_2 F_{21} \sigma_b T_2^4$$

Net energy exchange

$$Q_{12} = Q_1 - Q_2 \ = \varepsilon_1 \varepsilon_2 A_1 F_{12} \sigma_b T_1^4 - \varepsilon_1 \varepsilon_2 A_2 F_{21} \sigma_b T_2^4$$

$$A_1 F_{12} = A_2 F_{21}$$

$$Q_{12} = \varepsilon_1 \varepsilon_2 A_1 F_{12} \sigma_b \left(T_1^4 - T_2^4\right) = f_{12} A_1 F_{12} \sigma_b \left(T_1^4 - T_2^4\right)$$

Interchange factor , $f_{12} = \varepsilon_1 \varepsilon_2$

(2).One gray surface completely surrounded by another (Infinitely long cylindrical concentric cylinders)

- The large gray enclosure acts like a black body.
- It absorbs practically all radiation incident upon it and reflects negligibly small energy back to the small gray body.
- The entire radiation emitted by the small body would be intercepted by the outer large enclosure.

$$\therefore F_{12} = 1$$

Radiation calculations

Energy emitted by small body 1 and absorbed by large enclosure 2=

$$A_1 \varepsilon_1 \sigma_b T_1^4$$

Energy emitted by enclosure =

$$A_2 \varepsilon_2 \sigma_b T_2^4$$

Energy incident upon small body =

$$F_{21} A_2 \varepsilon_2 \sigma_b T_2^4$$

Energy absorbed by small body =

$$\alpha_1 F_{21} A_2 \varepsilon_2 \sigma_b T_2^4 = \varepsilon_1 \varepsilon_2 A_2 F_{21} \sigma_b T_2^4$$

Net exchange of energy =

$$Q_{12} = \alpha_1 A_1 \sigma_b T_1^4 - \varepsilon_1 \varepsilon_2 A_2 F_{21} \sigma_b T_2^4$$

Interchange Factor

If $T_1 = T_2$, $Q_{12} = 0$ and we get

$$A_1 = A_2 \varepsilon_2 F_{21}$$

$$\therefore Q_{12} = \varepsilon_1 A_1 \sigma_b \left(T_1^4 - T_2^4\right)$$

(so, $f_{12} = \varepsilon_1$)

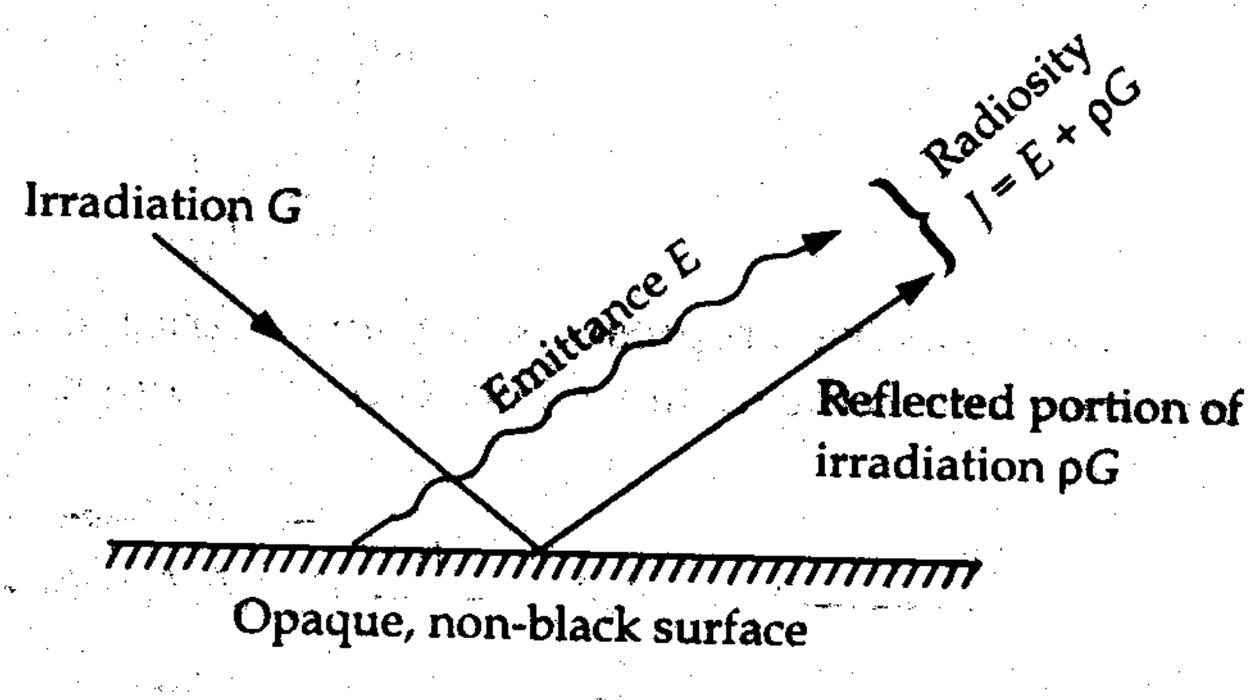
$$= f_{12} A_1 \sigma_b \left(T_1^4 - T_2^4\right)$$

ELECTRICAL NETWORK ANALOGY

Radiosity (J) indicates the total radiant energy leaving a surface per unit time per unit surface area. It comprises the original emittance from the surface plus the reflected portion of any radiation incident upon it.

Irradiation (G) denotes the total radiant energy incident upon a surface per unit time per unit area; some of it may be reflected to become a part of the radiosity of the surface.

Radiosity and Irradiation Concept



Radiosity and Irradiation Relation

The total radiant energy (J) leaving the surface is the sum of its original emittance (E) and the energy reflected (ρG) by it out of the irradiation (G) impinging on it.

Hence $J = E + \rho G$

$$= \varepsilon E_b + \rho G$$

E_b is the emissive power of black body at the same temperature

$\alpha + \rho = 1$ (opaque body)

$\rho = 1 - \alpha$

Radiosity and Irradiation Relation

From equ. we get,

$J = \varepsilon E_b + (1- \alpha)\, G$

$$\therefore G = \frac{J - \varepsilon E_b}{1 - \varepsilon}$$

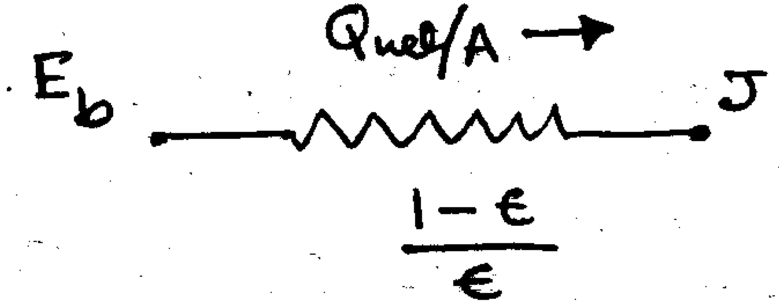
Now

$$\frac{Q_{net}}{A} = J - G = J - \frac{J - \varepsilon E_b}{1 - \varepsilon} = \frac{\varepsilon\left(E_b - J\right)}{1 - \varepsilon}$$

$$Q_{net} = \frac{A\varepsilon}{1 - \varepsilon}\left(E_b - J\right) = \frac{E_b - J}{\left(1 - \varepsilon\right)/ A\varepsilon}$$

Electrical Network Analogy

The above equation can be represented as electrical network as shown below



$$\frac{(1-\varepsilon)}{A\varepsilon}$$ is called surface resistance to radiation heat transfer.

Heat Transfer Between Non-Black Bodies

Heat transfer between two non-black surfaces is given by

$(Q_{1\text{-}2})_{net} = J_1A_1F_{12} - J_2A_2F_{21}$

J_1 and J_2 are the radiosities of surfaces 1 and 2.

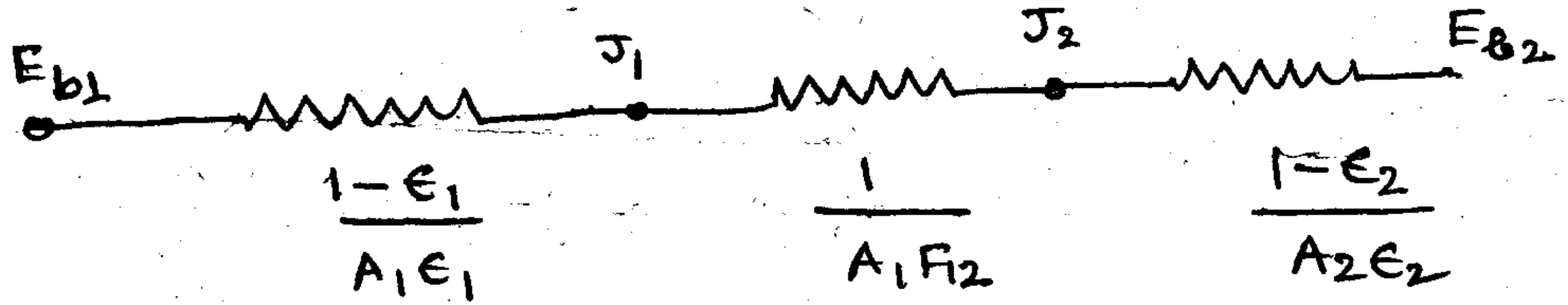
Also, $A_1F_{12} = A_2F_{21}$

$$\therefore (Q_{1-2})_{net} = (J_1 - J_2)A_1F_{12} = \frac{(J_1 - J_2)}{\dfrac{1}{A_1F_{12}}}$$

$\dfrac{1}{A_1F_{12}}$ is called space resistance.

Electrical Analogy Circuit

The final electrical analogy circuit for heat transfer between two non-black surfaces is drawn considering both surface resistance and space resistance as



Net Heat Transfer

$$\therefore \left(Q_{12}\right)_{net} = \frac{E_{b1} - E_{b2}}{\frac{\left(1-\varepsilon_1\right)}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{\left(1-\varepsilon_2\right)}{A_2\varepsilon_2}}$$

$$= \frac{A_1\sigma_b\left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{\left(1-\varepsilon_1\right)}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{\left(1-\varepsilon_2\right)}{\varepsilon_2} \times \frac{A_1}{A_2}}$$

$$= \left(F_g\right)_{12} A_1\sigma_b\left(T_1^{\,4} - T_2^{\,4}\right)$$

Gray body factor

$$\left(F_g\right)_{12} = \frac{1}{\frac{\left(1-\varepsilon_1\right)}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{\left(1-\varepsilon_2\right)}{\varepsilon_2} \times \frac{A_1}{A_2}}$$

Called Gray body factor

For radiant heat exchange between two black surfaces, the surface resistance becomes zero as

$$\varepsilon_1 = \varepsilon_2 = 1$$

And F_g becomes F_{12}, the shape factor only.So for black surfaces

$$\therefore Q_{12} = A_1 F_{12} \sigma_b \left(T_1^4 - T_2^4\right)$$

Special Cases

Two Infinite Parallel Planes:

Here, $F_{12} = F_{21} = 1$ and also $A_1 = A_2$

$$\left(F_g\right)_{12} = \frac{1}{\frac{\left(1-\varepsilon_1\right)}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{\left(1-\varepsilon_2\right)}{\varepsilon_2} \times \frac{A_1}{A_2}}$$

$$= \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Two Concentric Cylinders or Spheres

If the inner surface is surface 1, then $F_{12} = 1$

$$\left(F_g\right)_{12} = \frac{1}{\frac{\left(1-\varepsilon_1\right)}{\varepsilon_1} + 1 + \frac{\left(1-\varepsilon_2\right)}{\varepsilon_2} \times \frac{A_1}{A_2}}$$

Now, for concentric cylinders of equal length,

$$\frac{A_1}{A_2} = \frac{\pi d_1 l}{\pi d_2 l} = \frac{d_1}{d_2}$$

For concentric spheres,

$$\frac{A_1}{A_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

A small body in a large enclosure:

$F_{12} = 1$ $A_1 << A_2$ so $A_1/A_2 \longrightarrow 0$

$$\left(F_g\right)_{12} = \frac{1}{\frac{\left(1-\varepsilon_1\right)}{\varepsilon_1} + 1}$$

Practical example of this kind:

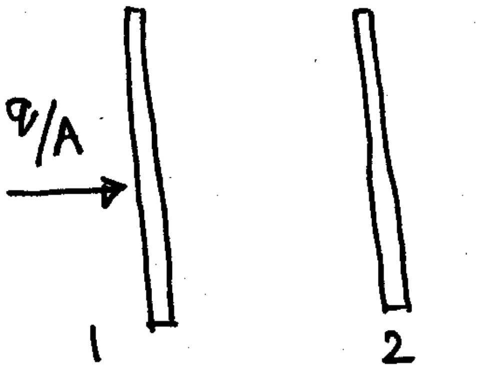
A pipe carrying steam in a large room

Radiation Shields

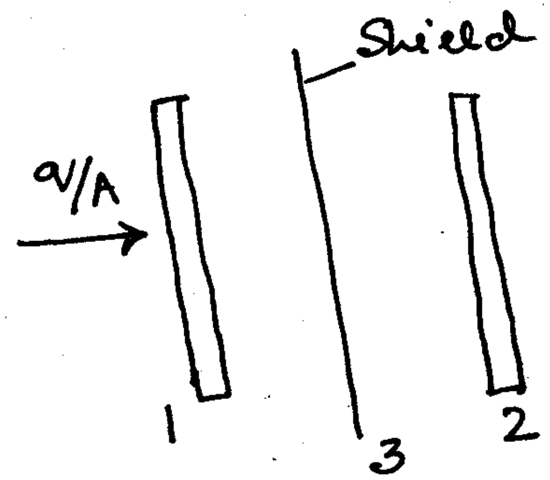
- One way of reducing radiant heat transfer between two particular surfaces is to use materials which are highly reflective.
- An alternative method is to use radiation shields between the hear exchange surfaces.
- The shields do not deliver or remove any heat from the overall system.
- **They only place another resistance in the heat flow path, so that the overall heat transfer is retarded.**

Single Radiation Shield

Consider two parallel infinite planes with and without shield.



Without Shield



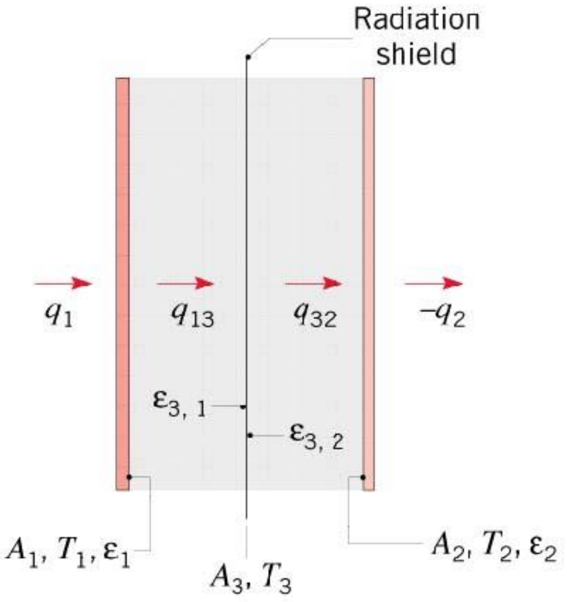
With Shield

Since the shield does not deliver or remove heat from the system, the heat transfer between plate 1 and the shield must be precisely the same, as that between the shield and plate 2, and this is the overall heat transfer.

Radiation Shields

- High reflectivity (low $\alpha = \varepsilon$) surface(s) inserted between two surfaces for which a reduction in radiation exchange is desired.

- Consider use of a single shield in a two-surface enclosure, such as that associated with large parallel plates:



Note that, although rarely the case, emissivities may differ for opposite surfaces of the shield.

Heat Transfer with shield

$$\therefore \left(\frac{q}{A}\right)_{1-3} = \left(\frac{q}{A}\right)_{3-2} = \frac{q}{A}$$

$$\frac{q}{A} = \frac{\sigma_b \left(T_1^4 - T_3^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{\sigma_b \left(T_3^4 - T_2^4\right)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

The only unknown in equation is the temperature of the shield T_3.

If the emissivity of all three surfaces are same, i.e., $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$, then

Heat Transfer with shield

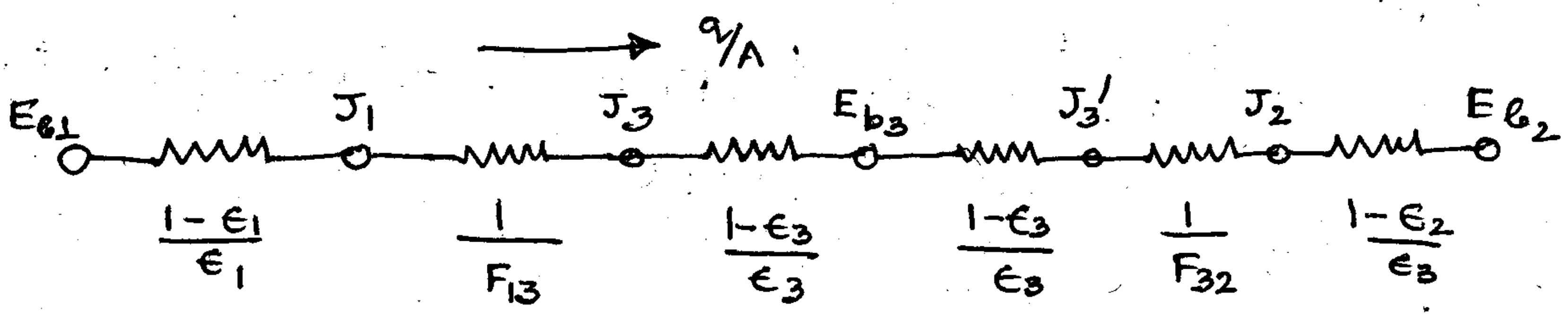
$$T_3^4 = \frac{1}{2}\left(T_1^4 + T_2^4\right)$$

The heat transfer is given by

$$\frac{q}{A} = \frac{1}{2}\,\frac{\sigma_b\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1}$$

As $\varepsilon_3 = \varepsilon_2$, the heat flow is just one-half of that which would be experienced if there is no shield present.

Equivalent circuit



When the emissivity of all surfaces are different, the overall heat transfer may be calculated most easily by using a series radiation network with appropriate number of elements as shown in the figure.

Multi Radiation shield

Consider n number of shields

Assume the emissivity of all the surfaces are same.

All the surface resistances will be same as the emissivity are same.

There will be two of these resistances for each shield and one for each heat transfer surface.

There will be (n+1) space resistances and these would all be unity since the radiation shape factors are unity for infinite parallel planes.

Multi Radiation Shield

Therefore, the total resistance in the network is

$$R_{(n\,shield)} = (2n+2)\frac{1-\varepsilon}{\varepsilon} + (n+1)(1) = \left(n+1\left(\frac{2}{\varepsilon}-1\right)\right)$$

The total resistance with no shield present

$$R_{(no\,shield)} = \frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1 = \frac{2}{\varepsilon} - 1$$

So, the resistance with shield is (n + 1) times the resistance without shield.

$$\therefore \left(\frac{q}{A}\right)_{with\ shields} = \frac{1}{n+1}\left(\frac{q}{A}\right)_{without\ shoeld}$$

Previous GATE Questions

SOLVED PROBLEMS

Q1:

Consider two black bodies with surfaces S1 (area = 1m^2) and S2 (area = 4 m^2). They exchange heat only by radiation, 40% of the energy emitted by S1 is received by S2. The fraction of energy emitted by S2 that is received by S1 is

(A) 0.05 (B) 0.1

(C) 0.4 (D) 0.6

$F_{12} A_1 = F_{21} A_2$

$A_1 = 1 m^2$

$A_2 = 4 m^2$

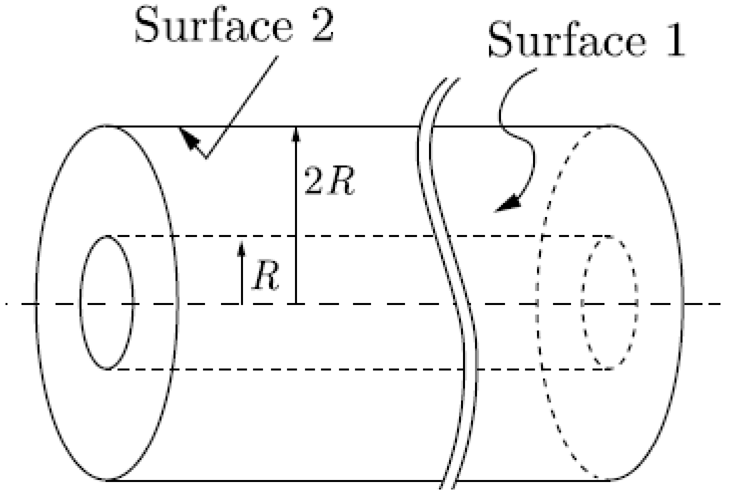
$F_{12} = 0.4$ given

$$F_{21} = \frac{F_{12} \times A_1}{A_2}$$

$$= 0.4 \times \frac{1}{4} = 0.1$$

Q2:

The view factor matrix for two infinitely long coaxial cylinders, shown in the figure below, is



(A) $\begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0.5 & 0.5 \\ 0 & 1 \end{bmatrix}$

View factor matrix is = $\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$

as Cylinder 1 can not see itself

$F_{11} = 0$

$F_{12} = 1$ (as Cylinder 1 is completely surrounded by 2)

$F_{12} A_1 = F_{21} A_2$

$\Rightarrow \frac{F_{12}}{F_{21}} = \frac{A_2}{A_1} = \frac{\pi (2R)L}{\pi R L} = 2$

$\Rightarrow F_{21} = \frac{F_{12}}{2} = \frac{1}{2} = 0.5$

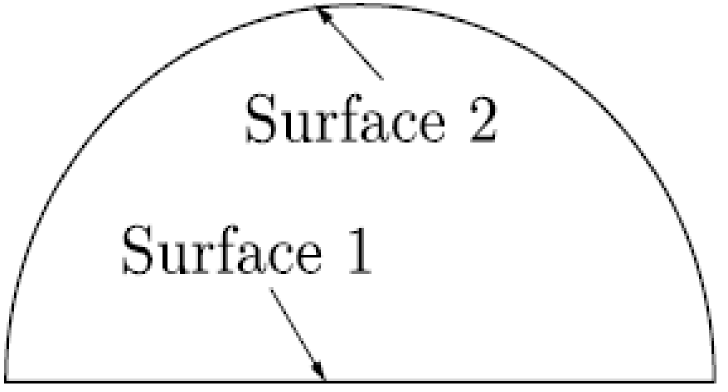
$F_{21} + F_{22} = 1 \Rightarrow F_{22} = 1 - 0.5 = 0.5$

$\therefore$ matrix is $\begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix}$

Q3:

A well-insulated hemispherical furnace (radius = 1m) is shown below

The self-view factor of radiation for the curved surface 2 is



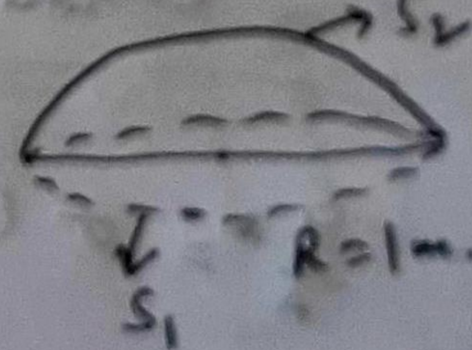
(A) 1/4 (B) 1/2

(C) 2/3 (D) 3/4

⇒ Per Rule

$F_{11} = 0$

$F_{12} = 1$



$F_{12} A_1 = F_{21} A_2$

$A_1 = \pi r^2$

$A_2 = 2\pi r^2$ (hemisphere)

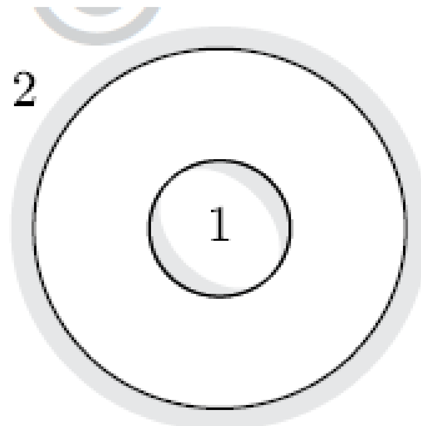
$\Rightarrow \frac{F_{12}}{F_{21}} = \frac{A_2}{A_1} \Rightarrow$

$\Rightarrow \frac{F_{21}}{F_{12}} = \frac{\pi r^2}{2\pi r^2}$

$\Rightarrow F_{21} = \frac{1}{2} = 0.5.$

Q4:

For the two long concentric cylinders with surface areas A_1 and A_2, the view factor F_{22} is given by



(A)0 (B) 1 (C) 1-(A_1/A_2) (D) A_1/A_2

$$A_1 F_{12} = A_2 F_{21}$$

$$\therefore F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{A_2}$$

$F_{11} = 0$

$F_{12} = 1$

$$F_{22} + F_{21} = 1$$

$$\Rightarrow F_{22} = 1 - F_{21} = 1 - \frac{A_1}{A_2}.$$

Q5:

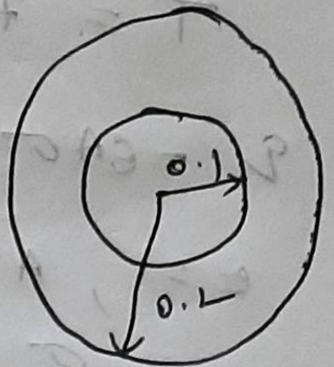
The space between two hollow concentric spheres of radii 0.1 m and 0.2 m is under vacuum. Exchange of radiation (uniform in all directions) occurs only between the outer surface (S_1) of the smaller sphere and the inner surface (S_2) of the larger sphere. The fraction (rounded off to the second decimal place) of the radiation energy leaving S_2, which reaches S_1 is ________

$A_1 F_{12} = A_2 F_{21}$

$F_{12} = 1$

$\therefore F_{21} = \frac{A_1}{A_2}$

$= \frac{4\pi(0.1)^2}{4\pi(0.2)^2} = 0.25$



Q6:

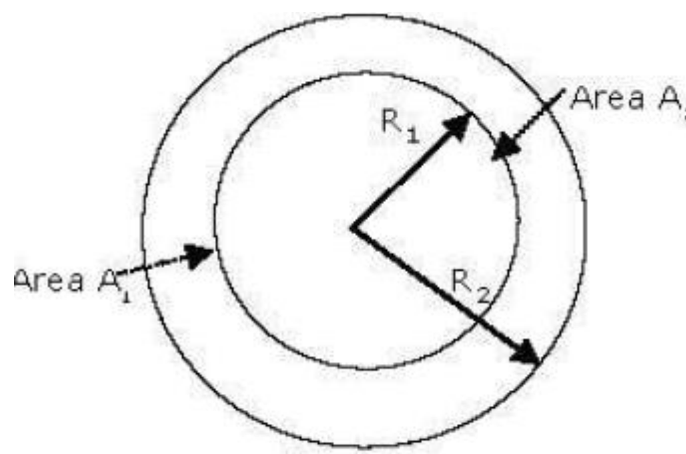
For the enclosure formed between two concentric spheres as shown below ($R2=2R1$), the fraction of radiation leaving the surface area A2 that strikes itself is

(A) 1/4

(B) 1/2

(C) $1/\sqrt{2}$

(D) 3/4

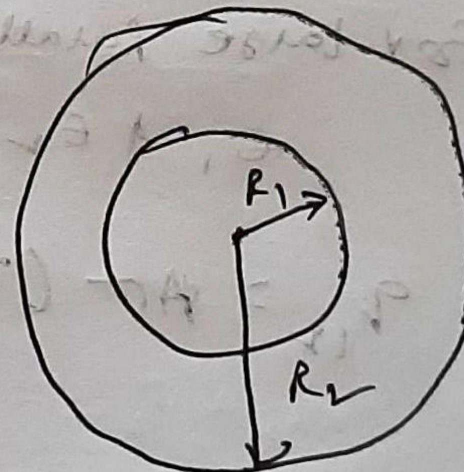


$R_2 = 2R_1$

$F_{11} + F_{12} = 1$

$F_{11} = 0$

$F_{12} = 1$



$A_1 F_{12} = A_2 F_{21}$

$$F_{21} = \frac{A_1}{A_2} F_{12}$$

$$= 1 \times \frac{4\pi R_1^2}{4\pi (2R_1)^2} = \frac{4}{16} = \frac{1}{4}$$

$F_{21} = 0.25 \Rightarrow F_{22} = 1 - F_{21} = 0.75$

$= 3/4$

Q7:

A small body with surface area A_1 having no concavities is surrounded by a large black surface of area A_2. Match the view factors:

F_{21}	A. 1
F_{22}	B. $1-(A_1/A_2)$
	C. A_1/A_2
	D. 0

Answer: F_{21} – C; F_{22} – B

Q8:

A sphere of radius, R_1 is enclosed in a sphere of radius R_2. The view (or shape) factor for radiative heat transfer of the outer sphere with respect to the inner sphere is:

A. 0

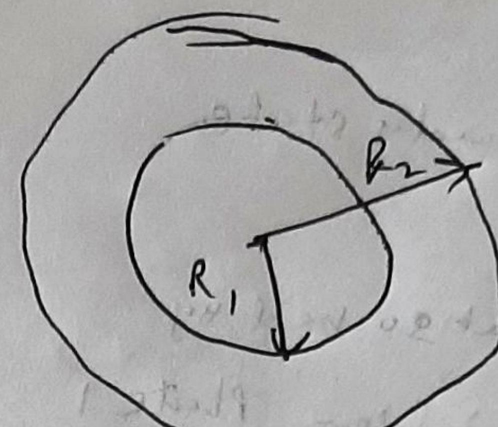
B. $R_2/(R_1+R_2)$

C. 1

D. $(R_1/R_2)^2$

as sphere 1 can see only shere '2'

$F_{12} = 1$



and $A_1 F_{12} = A_2 F_{21}$

$\Rightarrow (4\pi R_1^2) F_{12} = (4\pi R_2^2) F_{21}$

$\Rightarrow F_{21} = \left(\frac{R_1}{R_2}\right)^2$