CONTROL SYSTEMS

GATE CLASSES

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LESSON PLAN

- Introduction to Control systems
- Modeling of Physical systems
- Transfer function - Block diagram reduction Techniques
- Transfer function through Signal flow graph
- Time response of second order systems
- Steady state and Transient analysis
- Time-domain specifications and Static error coefficients.
- Routh-Hurwitz stability, Finding the range of K for stability
- Concepts of state, state variables and state model.
- Derivation of state model from transfer function
- State transition matrix, Properties, determination of STM
- Conversion from SS to TF

DAY-4

- Steady state error and Static error coefficients.
- Routh-Hurwitz stability
- Finding the range of K for stability


Forget
the mistake.
Remember
the lesson.
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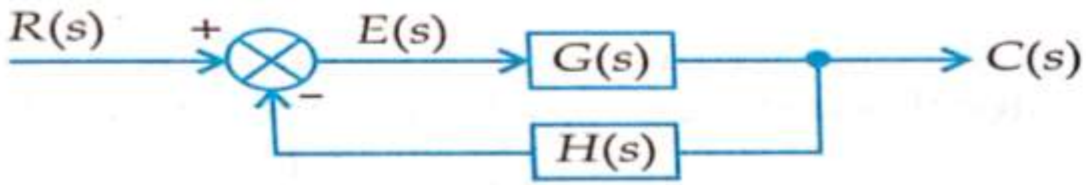
Steady state error

The steady-state error is the difference between the input and output of the system during steady state. For accuracy the steady state error should be minimum.



$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$$

$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

The steady state error of the system is obtained by applying final value theorem.

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s.E(s)$$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)H(s)}$$

For unity feedback system $H(s) = 1$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)}$$

Static Error Constants

- The static error constants are figures of merit of control systems. **The higher the constants, the smaller the steady-state error.**
- In a mechanical system, the output may be the position, velocity, acceleration.
- We shall call the output in a mechanical translational system as “position,”(or displacement) the rate of change of the output “velocity,” and so on.
- This means that in a system “position” represents the output , “velocity” represents the rate of change of the output and so on.

Static Position Error Constant (K_p)

- The steady-state error of the system for a unit-step input is

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{\cancel{s}}{1 + G(s)} \frac{1}{\cancel{s}}$$

$$= \frac{1}{1 + G(0)}$$

- The static position error constant K_p is defined by

$$K_p = \lim_{s \to 0} G(s) = G(0)$$

- Thus, the steady-state error in terms of the static position error constant K_p is given by

$$e_{ss} = \frac{1}{1 + K_p}$$

Static Position Error Constant (K_p)

- For a Type 0 system

$$K_p = \lim_{s \to 0} \frac{K(T_a s + 1)(T_b s + 1)\cdots}{(T_1 s + 1)(T_2 s + 1)\cdots} = K$$

- For Type 1 or higher order systems

$$K_p = \lim_{s \to 0} \frac{K(T_a s + 1)(T_b s + 1)\cdots}{s^N(T_1 s + 1)(T_2 s + 1)\cdots} = \infty, \qquad \text{for } N \geq 1$$

- For a unit step input the steady state error **e_{ss}** is

$$e_{ss} = \frac{1}{1 + K}, \qquad \text{for type 0 systems}$$

$$e_{ss} = 0, \qquad \text{for type 1 or higher systems}$$

Static Velocity Error Constant (K_v)

- The steady-state error of the system for a unit-ramp input is

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{R(s)}{1+G(s)} \qquad e_{ss} = \lim_{s\to 0} \frac{s}{1+G(s)} \frac{1}{s^2}$$

$$= \lim_{s\to 0} \frac{1}{sG(s)}$$

- The static velocity error constant K_v is defined by

$$K_v = \lim_{s\to 0} sG(s)$$

- Thus, the steady-state error in terms of the static velocity error constant K_v is given by

$$e_{ss} = \frac{1}{K_v}$$

Static Velocity Error Constant (K_v)

- For a Type 0 system

$$K_v = \lim_{s \to 0} \frac{sK(T_a s + 1)(T_b s + 1)\cdots}{(T_1 s + 1)(T_2 s + 1)\cdots} = 0$$

- For Type 1 systems

$$K_v = \lim_{s \to 0} \frac{sK(T_a s + 1)(T_b s + 1)\cdots}{s(T_1 s + 1)(T_2 s + 1)\cdots} = K$$

- For type 2 or higher order systems

$$K_v = \lim_{s \to 0} \frac{sK(T_a s + 1)(T_b s + 1)\cdots}{s^N(T_1 s + 1)(T_2 s + 1)\cdots} = \infty, \qquad \text{for } N \geq 2$$

Static Velocity Error Constant (K_v)

- For a ramp input the steady state error **e_{ss}** is

$$e_{ss} = \frac{1}{K_v} = \infty, \qquad \text{for type 0 systems}$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K}, \qquad \text{for type 1 systems}$$

$$e_{ss} = \frac{1}{K_v} = 0, \qquad \text{for type 2 or higher systems}$$

Static Acceleration Error Constant (K_a)

- The steady-state error of the system for parabolic input is

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{s}{1 + G(s)} \frac{1}{s^3} = \frac{1}{\lim_{s\to 0} s^2 G(s)}$$

- The static acceleration error constant K_a is defined by

$$K_a = \lim_{s\to 0} s^2 G(s)$$

- Thus, the steady-state error in terms of the static acceleration error constant K_a is given by

$$e_{ss} = \frac{1}{K_a}$$

Static Acceleration Error Constant (K_a)

- For a Type 0 system

$$K_a = \lim_{s\to 0} \frac{s^2K(T_as + 1)(T_bs + 1)\cdots}{(T_1s + 1)(T_2s + 1)\cdots} = 0$$

- For Type 1 systems

$$K_a = \lim_{s\to 0} \frac{s^2K(T_as + 1)(T_bs + 1)\cdots}{s(T_1s + 1)(T_2s + 1)\cdots} = 0$$

- For type 2 systems

$$K_a = \lim_{s\to 0} \frac{s^2K(T_as + 1)(T_bs + 1)\cdots}{s^2(T_1s + 1)(T_2s + 1)\cdots} = K$$

- For type 3 or higher order systems

$$K_a = \lim_{s\to 0} \frac{s^2K(T_as + 1)(T_bs + 1)\cdots}{s^N(T_1s + 1)(T_2s + 1)\cdots} = \infty, \qquad \text{for } N \geq 3$$

Static Acceleration Error Constant (K_a)

- For a parabolic input the steady state error **e_{ss}** is

$$e_{ss} = \infty, \quad \text{for type 0 and type 1 systems}$$

$$e_{ss} = \frac{1}{K}, \quad \text{for type 2 systems}$$

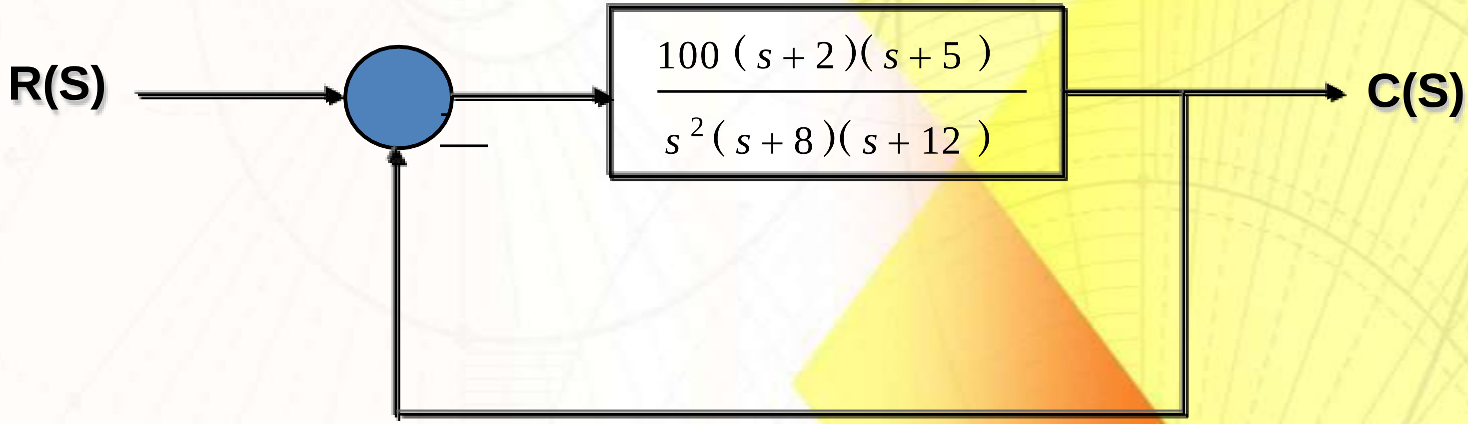
$$e_{ss} = 0, \quad \text{for type 3 or higher systems}$$

Summary of steady state error

	Step Input $r(t) = 1$	Ramp Input $r(t) = t$	Acceleration Input $r(t) = \frac{1}{2}t^2$
Type 0 system	$\frac{1}{1+K}$	∞	∞
Type 1 system	0	$\frac{1}{K}$	∞
Type 2 system	0	0	$\frac{1}{K}$

Example

For the system shown in figure below evaluate the static error constants and find the expected steady state errors for the standard step, ramp and parabolic inputs.



Example

$$G(s) = \frac{100(s+2)(s+5)}{s^2(s+8)(s+12)}$$

$$K_p = \lim_{s \to 0} G(s)$$

$$K_p = \lim_{s \to 0} \left(\frac{100(s+2)(s+5)}{s^2(s+8)(s+12)} \right)$$

$$K_p = \infty$$

$$K_v = \lim_{s \to 0} sG(s)$$

$$K_v = \lim_{s \to 0} \left(\frac{100 s(s+2)(s+5)}{s^2(s+8)(s+12)} \right)$$

$$K_v = \infty$$

$$K_a = \lim_{s \to 0} s^2 G(s)$$

$$K_a = \lim_{s \to 0} \left(\frac{100 s^2(s+2)(s+5)}{s^2(s+8)(s+12)} \right)$$

$$K_a = \left(\frac{100(0+2)(0+5)}{(0+8)(0+12)} \right) = 10.4$$

Example

$K_p = \infty$ $K_v = \infty$ $K_a = 10.4$

$$e_{ss} = \frac{1}{1 + K_p} = 0$$

$$e_{ss} = \frac{1}{K_v} = 0$$

$$e_{ss} = \frac{1}{K_a} = 0.09$$

The open loop transfer function of unity feedback system is given by

$$G(s) = \frac{50}{(1+0.1s)(s+10)}$$

Determine the static error coefficients K_p, K_v and K_a.

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$= \lim_{s\to 0} \frac{50}{(1+0.1s)(s+10)} = 5$$

$$K_v = \lim_{s\to 0} s\,G(s)H(s) = \lim_{s\to 0} s\cdot\frac{50}{(1+0.1s)(s+10)} = 0$$

$$K_a = \lim_{s\to 0} s^2 \frac{50}{(1+0.1s)(s+10)} = 0$$

Determine the step, ramp and parabolic error coefficients.

$$G(s) = \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$$

$$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$$

$$K_p = \infty \qquad \boxed{K_p = \infty}$$

$$K_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} s\cdot\frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$$

$$K_v = \infty \qquad \boxed{K_v = \infty}$$

$$K_a = \lim_{s\to 0} s^2G(s)H(s) = \lim_{s\to 0} s^2\cdot\frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$$

$$= 10 \qquad \boxed{K_a = 10}$$

Determine the type of the system and compute the error constant K_v

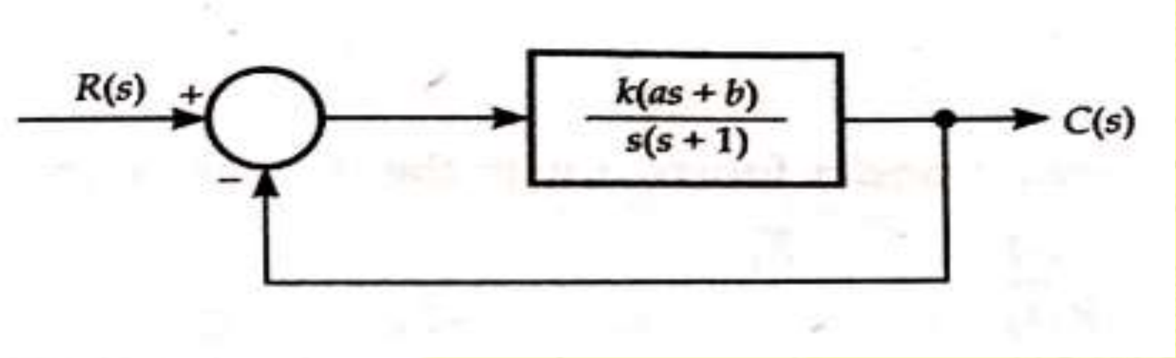
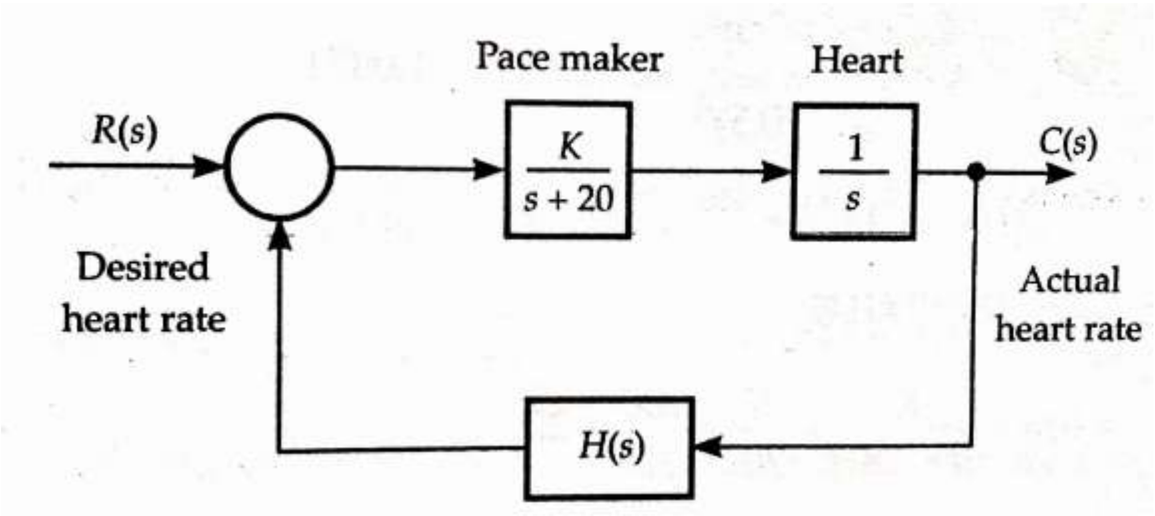


Diagram shown for a pacemaker to control the heart rate.

Determine steady state error for unit ramp input assuming K=400
Determine the value of K if error for ramp input is 0.02



The open-loop transfer function of a unity feedback system is given by

$$G(s) = \frac{K}{s(sT+2)}$$

(a) By what factor should the gain K be multiplied to increase the damping ratio from 0.15 to 0.60.

(b) Determine the factor by which the time constant T be should be multiplied to reduce the damping ratio from 0.8 to 0.4.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{K}{s(sT+2)}}{1+\frac{K}{s(sT+2)}} = \frac{K}{s^2T+2s+K} = \frac{(K/T)}{s^2+\frac{2}{T}s+\frac{K}{T}}$$

Comparing with the standard characteristic equation

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{K/T}{s^2+\frac{2}{T}s+\frac{K}{T}}$$

$$\omega_n^2 = \frac{K}{T}, \quad \omega_n = \sqrt{\frac{K}{T}}$$

$$2\zeta\omega_n = \frac{2}{T}$$

(*a*) Let $\zeta = \zeta_1$ and $K = K_1$ for $\zeta = 0.15$ and $\zeta = \zeta_2$, $K = K_2$ for $\zeta = 0.60$

Therefore,
$$\zeta_1 = \frac{1}{\sqrt{K_1 T}}$$

$$\zeta_2 = \frac{1}{\sqrt{K_2 T}}$$

$$\frac{\zeta_1}{\zeta_2} = \sqrt{\frac{K_2 T}{K_1 T}} = \frac{0.15}{0.60} = \frac{1}{4}$$

Therefore,
$$\frac{K_2}{K_1} = \frac{1}{16}$$

$$K_2 = \frac{1}{16} K_1$$

Thus, the gain should be multiplied by factor $\frac{1}{16}$ to increase the damping ratio

(*b*) Let
$$\zeta_1 = \frac{1}{\sqrt{T_1 K}} \quad \text{for } \zeta = 0.15$$

and
$$\zeta_2 = \frac{1}{\sqrt{T_2 K}} \quad \text{for } \zeta = 0.60$$

$$\frac{\zeta_1}{\zeta_2} = \sqrt{\frac{T_2 K}{T_1 K}} = \frac{0.8}{0.4} = 2$$

Therefore,
$$\frac{T_2}{T_1} = 4$$

$$T_2 = 4 T_1$$

Hence, the time constant T should be multiplied by a factor 4 to reduce the damping ratio from 0.8 to 0.4.

The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{108}{s^2(s+4)(s^2+3s+12)}$$

Find the static error coefficients and steady state error of the system when subjected to an input given by

$$r(t) = 2 + 5t + 2t^2$$

$$K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{108}{s^2(s+4)(s^2+3s+12)} = \infty$$

$$K_v = \lim_{s\to 0} s\cdot G(s) = \lim_{s\to 0} s\cdot \frac{108}{s^2(s+4)(s^2+3s+12)} = \infty$$

$$K_a = \lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} s^2\cdot \frac{108}{s^2(s+4)(s^2+3s+12)} = \frac{108}{48}$$

$$r(t) = 2 + 5t + 2t^2$$

$$R(s) = \frac{2}{s} + \frac{5}{s^2} + \frac{4}{s^3}$$

$$e_{ss} = \frac{R_1}{1+K_p} + \frac{R_2}{K_v} + \frac{R_3}{K_a}$$

$$= \frac{2}{1+\infty} + \frac{5}{\infty} + \frac{4\times 48}{108} = 1.77$$

$$\boxed{e_{ss} = 1.77}$$

Generalized error coefficients

The dynamic error coefficient method is a generalized method to include inputs of any arbitrary functions of time.

For a unity feedback system,

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = F(s)$$

The function $F(s)$ can be expanded as a power series in s by Taylor's series expansion as

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = C_0 + C_1 s + C_2 s^2 + C_3 s^3 + \ldots$$

where C_0, C_1, C_2, C_3, ... are defined to be ***dynamic error coefficient*** or ***generalized error coefficients***.

Therefore, $$E(s) = C_0 R(s) + C_1 sR(s) + \frac{C_2}{2!} s^2 R(s) + \frac{C_3}{3!} s^3 R(s) + \ldots$$

Taking the inverse Laplace transform, the dynamic error is given by

$$e(t) = C_0 r(t) + C_1 \dot{r}(t) + \frac{C_2}{2!} \ddot{r}(t) + \ldots$$

The generalised error coefficients are calculated as follows

$$C_0 = \lim_{s \to 0} F(s)$$

$$C_1 = \lim_{s \to 0} \frac{d}{ds} F(s)$$

$$C_2 = \lim_{s \to 0} \frac{d^2}{ds^2} F(s)$$

Correlation between static and dynamic error constants

$$C_0 = \frac{1}{1 + K_p}$$

$$C_1 = \frac{1}{K_v}$$

$$C_2 = \frac{1}{K_a}$$

For the closed-loop system with

$$G(s) = \frac{1}{s+5} \text{ and } H(s) = 5,$$

calculate the generalized error coefficients and find error series.

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$$

$$E(s) = F(s)R(s)$$

$$F(s) = \frac{1}{1+G(s)H(s)}$$

$$F(s) = \frac{1}{1+\frac{1}{s+5}\cdot 5} = \frac{s+5}{s+10}$$

The error coefficients are

$$C_0 = \lim_{s\to 0} F(s) = \lim_{s\to 0} \frac{s+5}{s+10} = 0.5$$

$$C_1 = \lim_{s\to 0} \frac{d}{ds} F(s) = \lim_{s\to 0} \frac{(s+10)-(s+5)}{(s+10)^2}$$

$$= \lim_{s\to 0} \frac{5}{(s+10)^2} = 0.05$$

$$C_2 = \lim_{s\to 0} \frac{d^2 F(s)}{ds^2}$$

$$= \lim_{s\to 0} \frac{d}{ds} \frac{5}{(s+10)^2}$$

$$= \lim_{s\to 0} \frac{5(-2)}{(s+10)^3} = -0.01$$

Therefore, the dynamic error is

$$e(t) = C_0\, r(t) + C_1 \frac{d}{dt} r(t) + \frac{C_2}{2!} \frac{d^2}{dt^2} r(t) + \ldots$$

$$= 0.5\; r(t) + 0.05\; \dot{r}(t) - \frac{0.01}{2}\; \ddot{r}(t) + \ldots$$

Methods to improve Time response

It is necessary for a control system to meet certain specifications for better performance.
This depends upon the task of a control system is expected to do.
Generally system performance can be improved by any of the following control methods

1. Proportional control (P)
2. Proportional and derivative control (PD)
3. Proportional plus Integral control (PI)
4. Proportional plus Integral plus Derivative control(PID)

A controller is a device which is introduced in feed back or forward path of a system to control the steady state and transient response as per the requirement