AP Government GATE Online Classes

Heat Transfer

Day-3 (28.05.2020)

Dr. R. Srikanth
Professor
ANITS-Visakhapatnam

HEAT TRANSFER TO FLUIDS WITHOUT PHASE CHANGE

CONVECTION

FORCED AND NATURAL

CONVECTION

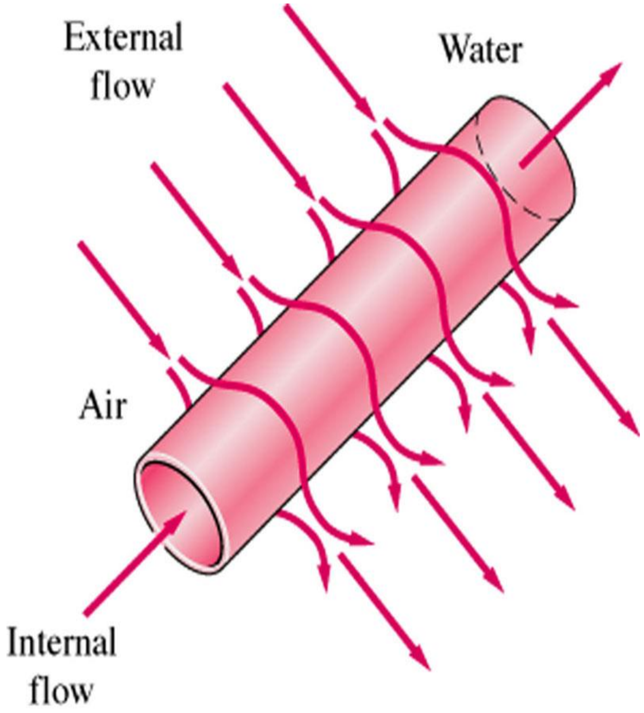
Mechanism of heat transfer through a fluid in the presence of bulk fluid motion

- **Natural (free) Convection**
- **Forced Convection**

(depending on how the fluid motion is initiated)

CLASSIFICATION OF FLUID FLOWS

- **Viscous-inviscid**
- **Internal flow- External flow**
- **Open-closed channel**
- **Compressible- Incompressible**
- **Laminar- Turbulent**
- **Natural- Forced**
- **Steady- Unsteady**
- **One-,two-,three-dimensional**



VISCOSITY

When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer.

internal resistance to flow

- cohesive forces between the molecules in liquid
- molecular collisions in gases.

Viscous flows: viscous effects are significant

Inviscid flow regions: viscous forces are negligibly small compared to inertial or pressure forces.

→ measure of stickness or resistance to deformation

1. **Kinematic viscosity**
2. **Dynamic viscosity**

VISCOSITY DEPENDS ON

- **TEMPERATURE**
- **PRESSURE**

For liquids dependence of pressure is negligible

For gases kinematic viscosity depends on pressure since its relation to density

Air at 20°C and 1 atm:

$$\mu = 1.83 \times 10^{-5} \text{ kg/m} \cdot \text{s}$$

$$\upsilon = 1.52 \times 10^{-5} \text{ m}^2\text{/s}$$

Air at 20°C and 4 atm:

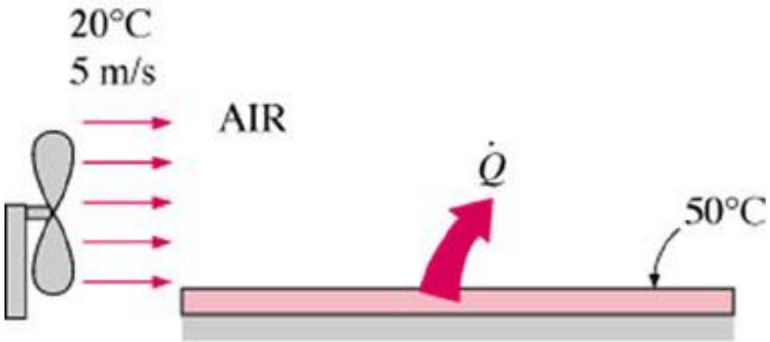
$$\mu = 1.83 \times 10^{-5} \text{ kg/m} \cdot \text{s}$$

$$\upsilon = 0.380 \times 10^{-5} \text{ m}^2\text{/s}$$

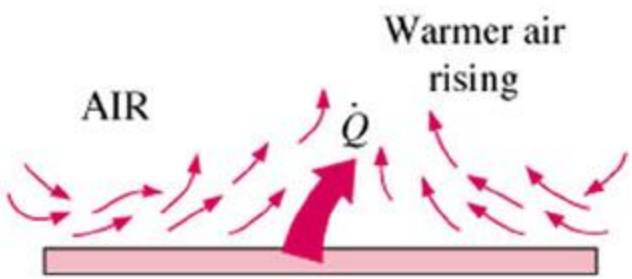
μ Dynamic viscosity (kg/m.s or poise)

$$\nu = \frac{\mu}{\rho}$$

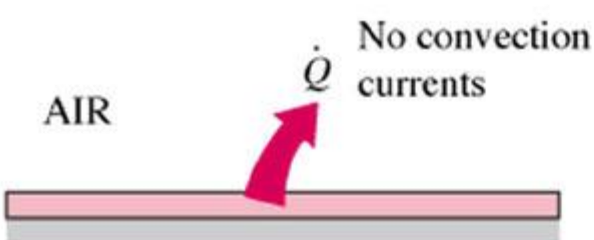
ν Kinematic viscosity, m^2/s or stroke



(a) Forced convection



(b) Free convection



(c) Conduction

Convection heat transfer

- Dynamic viscosity
- Thermal conductivity
- Density
- Specific heat
- Fluid velocity
- Geometry
- Roughness
- Type of fluid flow

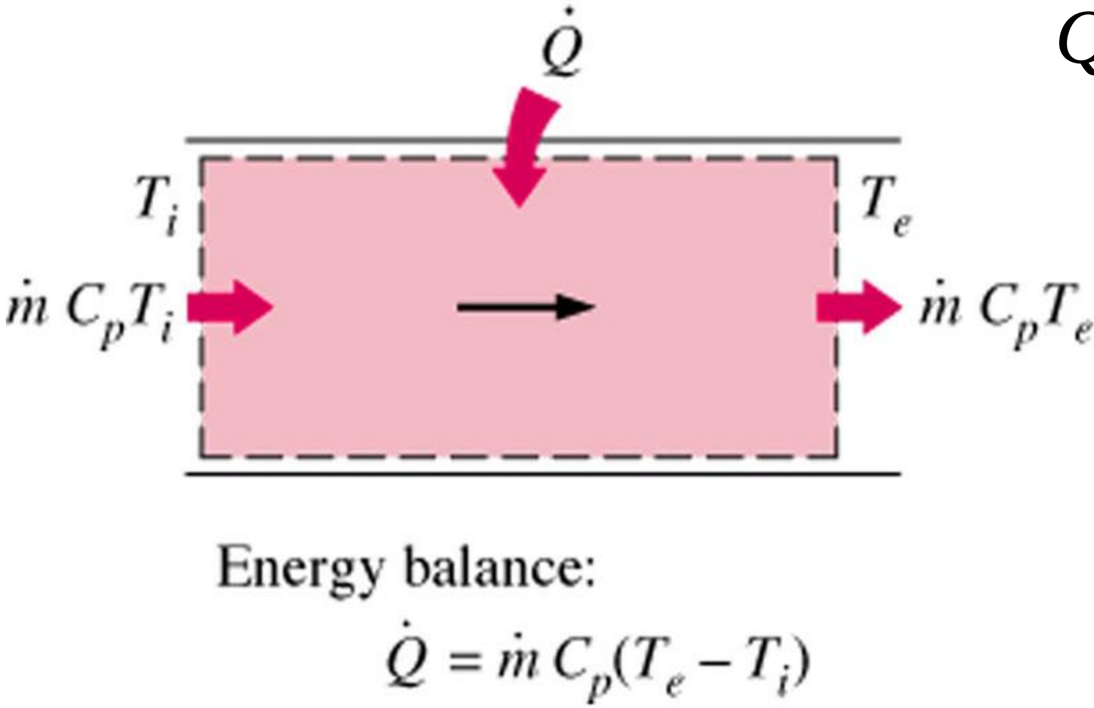
NEWTON'S LAW OF COOLING

$$\dot{Q}_{conv} = hA_S(T_S - T_\infty) \quad \text{(W)}$$

h Convection heat transfer coefficient ($W/m^2.^0C$)

The rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference

GENERAL THERMAL ANALYSIS



$$\dot{Q}_{conv} = hA_S\,(T_S - T_\infty)$$

$$\dot{Q} = \dot{m}\, C_p\,(T_e - T_i)$$

The Convection Problem

- **“h” is often the controlling parameter in heat transfer problems involving fluids; knowing its value accurately is important**
- **h can be obtained by**
  - theoretical derivation (difficult)
  - direct measurement (time-consuming)
  - empirical correlation (most common)
- **Theoretical derivation is difficult because**
  - h is dependent upon many parameters
  - it involves solving several PDEs
  - it usually involves turbulent fluid flow, for which no unified modeling approach exists

FORCED CONVECTION

Boundary Layers Introduction

- A boundary layer is a thin region in the fluid adjacent to a surface where velocity, temperature and/or concentration gradients normal to the surface are significant.
- Typically, the flow is predominantly in one direction.
- As the fluid moves over a surface, a velocity gradient is present in a region known as the velocity boundary layer, $\delta(x)$.
- Likewise, a temperature gradient forms ($T_{\infty} \neq T_s$) in the thermal boundary layer, $\delta_t(x)$,
- Flat Plate Boundary Layer is an hypothetical standard for initiation of basic analysis.

Boundary Layers: Physical Features

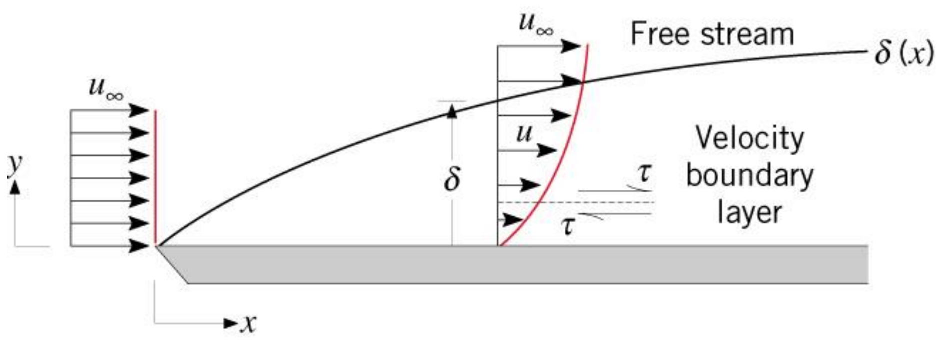
- Velocity Boundary Layer

  – A consequence of viscous effects associated with relative motion between a fluid and a surface.

  – A region of the flow characterized by shear stresses and velocity gradients.

  – A region between the surface and the free stream whose thickness δ increases in the flow direction.

  – Manifested by a surface shear stress τ_s that provides a drag force, F_D .



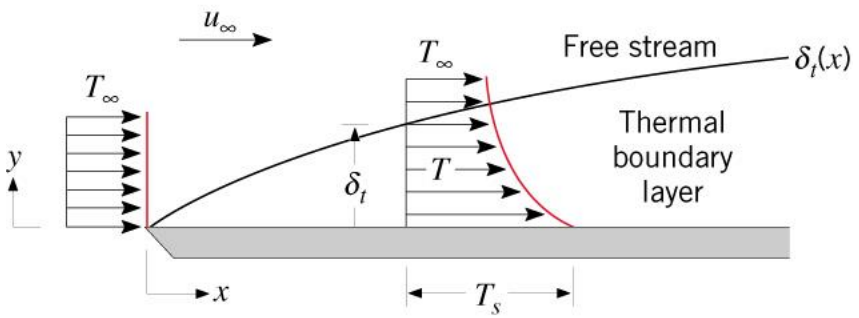
$$\delta \to \frac{u(y)}{u_\infty} = 0.99$$

$$\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$F_D = \int_{A_s} \tau_s dA_s$$

- Thermal Boundary Layer

  - A consequence of heat transfer between the surface and fluid.
  - A region of the flow characterized by temperature gradients and heat fluxes.
  - A region between the surface and the free stream whose thickness δ_t increases in the flow direction.
  - Manifested by a surface heat flux q''_s and a convection heat transfer coefficient h .



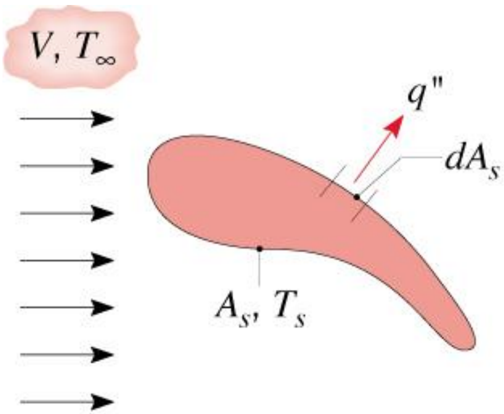
$$\delta_t \rightarrow \frac{T_s - T(y)}{T_s - T_\infty} = 0.99$$

$$q''_s = -k_f \frac{\partial T}{\partial y}\Big|_{y=0}$$

$$h \equiv \frac{-k_f \partial T / \partial y|_{y=0}}{T_s - T_\infty}$$

Distinction between Local and Average Heat Transfer Coefficients

- Local Heat Flux and Coefficient:



$$q'' = h\left(T_s - T_\infty\right)$$

- Average Heat Flux and Coefficient for a Uniform Surface Temperature:

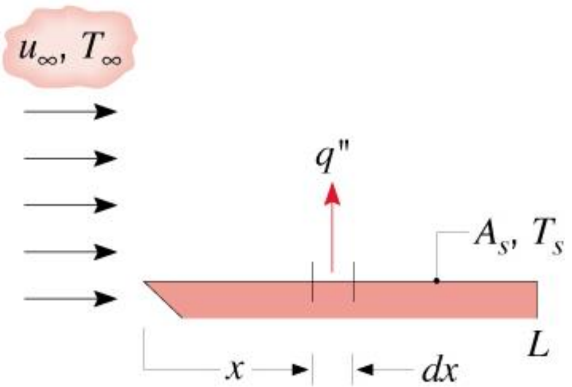
$$q = \bar{h} A_s \left(T_s - T_\infty\right)$$

$$q = \int_{A_s} q'' dA_s = \left(T_s - T_\infty\right) \int_{A_s} h dA_s$$

$$\bar{h} = \frac{1}{A_s} \int_{A_s} h dA_s$$

- For a flat plate in parallel flow:

$$\bar{h} = \frac{1}{L} \int_o^L h dx$$



- **LAMINAR FLOW**

Smooth streamlines

Highly- ordered motion

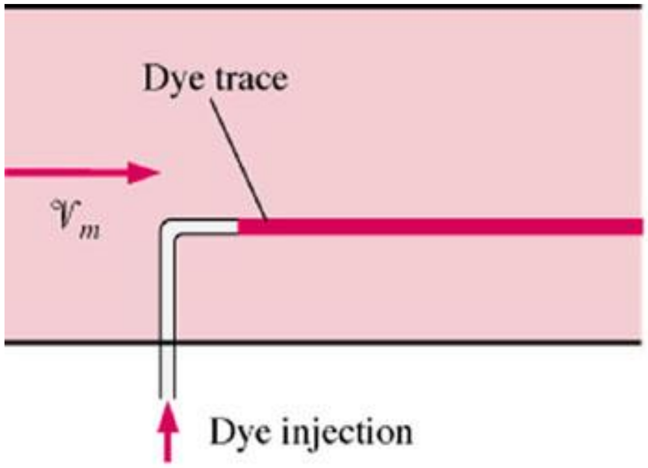
(highly viscous fluids in small pipes)

- **TURBULENT FLOW**

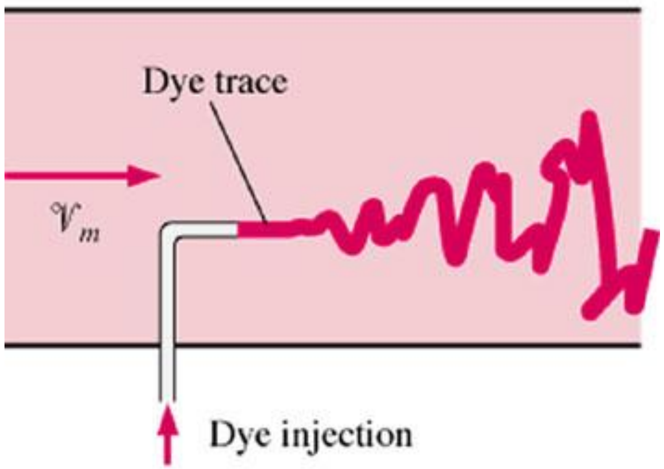
Velocity fluctuations

Highly-disordered motion

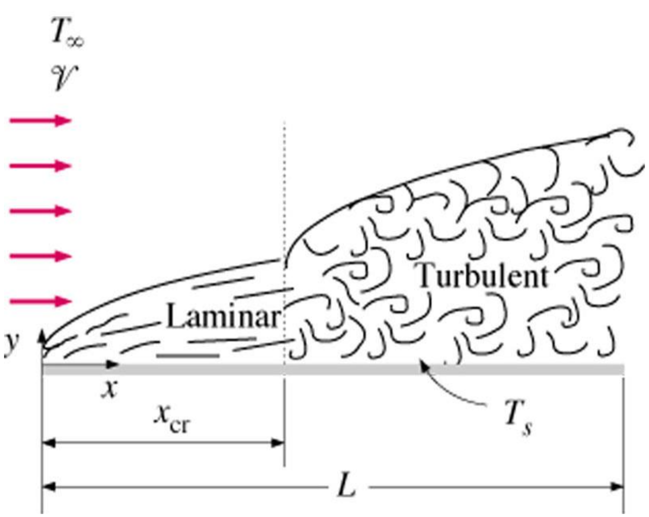
- **TRANSITIONAL FLOW**



(a) Laminar flow



(b) Turbulent flow



REYNOLDS NUMBER

Flow Regime:

Geometry

Surface roughness

Flow velocity

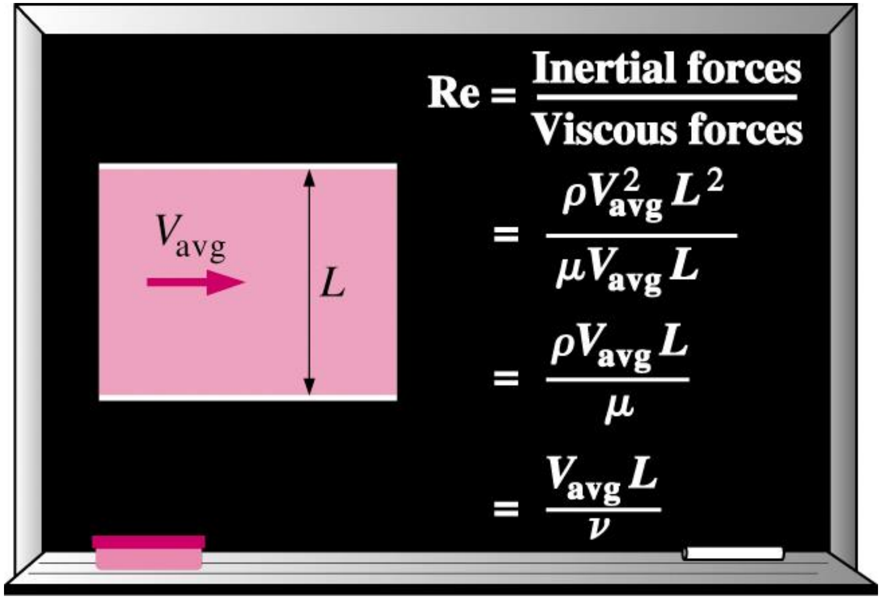
Surface temperature
type of fluid

Ratio of the inertial forces to viscous forces in the fluid

$$\mathrm{Re} = \frac{\upsilon_m D}{v} = \frac{\rho \upsilon_m D}{\mu}$$

υ_m	Mean flow velocity
D	Characteristic length of the geometry
$v = \mu / \rho$	Kinematic viscosity

Definition of Reynolds number



- Critical Reynolds number (Re_{cr}) for flow in a round pipe

  $Re < 2100 \Rightarrow$ laminar

  $2100 \le Re \le 4000 \Rightarrow$ transitional

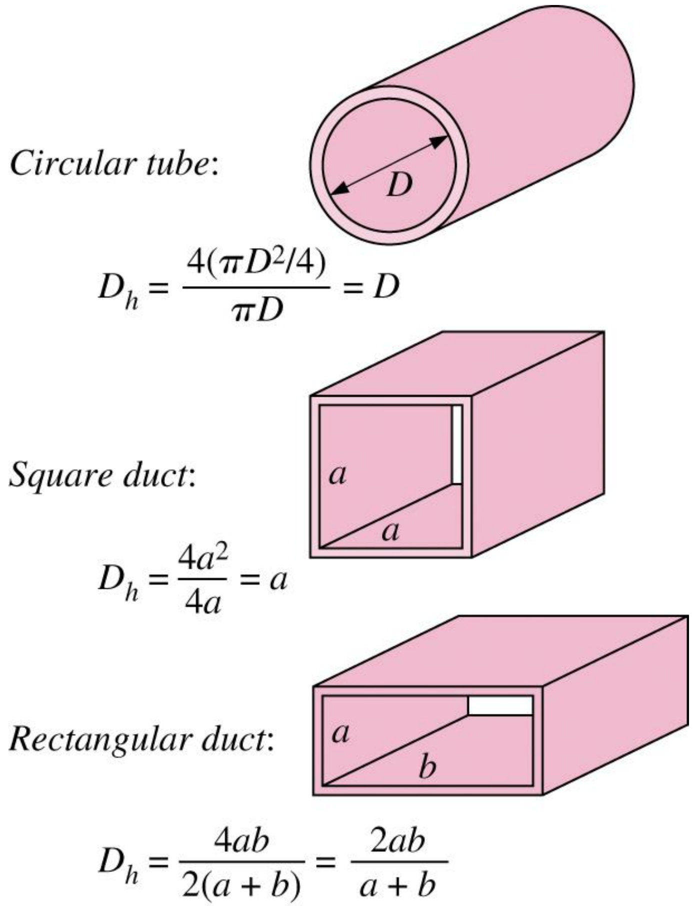
  $Re > 4000 \Rightarrow$ turbulent

- Note that these values are approximate.
- For a given application, Re_{cr} depends upon
  - Pipe roughness
  - Vibrations
  - Upstream fluctuations, disturbances (valves, elbows, etc. that may disturb the flow)

Turbulence

- **Turbulence represents an unsteady flow, characterized by random velocity, pressure, and temperature fluctuations in the fluid due to small-scale *eddies***
- **Turbulence occurs when the Reynolds number reaches some critical value, determined by the particular flow geometry**
- **Turbulence is typically modeled with *eddy diffusivities* for mass, momentum, and heat**
- **Turbulent flow increases viscous drag, but may actually reduce form drag in some instances**
- **Turbulent flow is advantageous in the sense of providing higher *h*-values, leading to higher convection heat transfer rates**

HYDRAULIC DIAMETER



- For <u>non-round</u> pipes,
- the hydraulic diameter $D_h = 4A_c/P$

  A_c = cross-section area

  P = wetted perimeter

PRANDTL NUMBER

- Boundary layer theory

$$\Pr = \frac{\mu C_p}{k}$$

$$\Pr = \frac{molecular\ \ diffusivity\ \ of\ \ momentum}{molecular\ \ diffusivity\ \ of\ \ heat} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

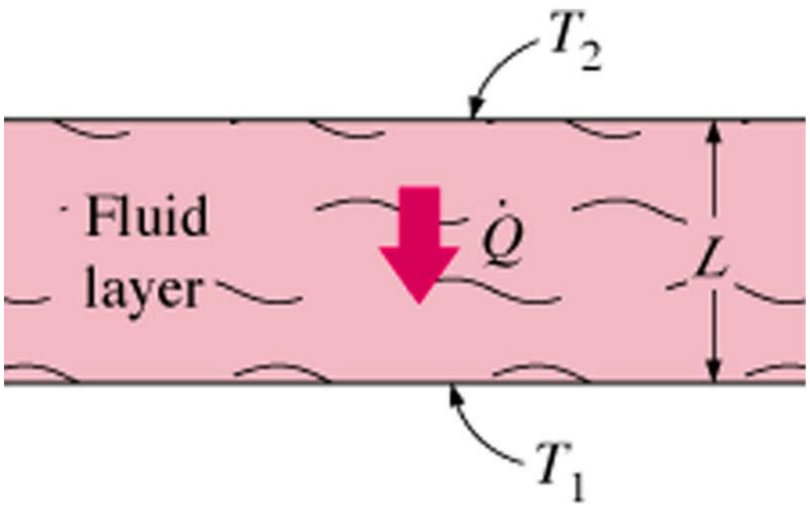
Prandtl number ($\mu cp/k$) represents the ratio of momentum diffusion to heat diffusion in the boundary layer.

small Pr: relatively large thermal boundary layer thickness ($\delta t > \delta$)

large Pr: relatively small thermal boundary layer thickness ($\delta t < \delta$)

NUSSELT NUMBER

(Dimensionless number)



$$\Delta T = T_2 - T_1$$

$$Nu = \frac{hL_c}{k}$$

$$\dot{q}_{cond} = k\frac{\Delta T}{L}$$

$$\dot{q}_{conv} = h\Delta T$$

$$\frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h\Delta T}{k\Delta T / L} = \frac{hL}{k} = Nu$$

$$Nu = \frac{hL}{k_{fluid}} = \frac{\frac{L}{k_{fluid}}}{\frac{1}{h}} = \frac{\text{Conduction resistance}}{\text{Convection resistance}}$$

The Nusselt Number

- **The normalized BL equations indicate that**

$$T^* = f\left(x^*, y^*, \mathrm{Re}, \mathrm{Pr}, \frac{dp^*}{dx^*}\right)$$

- **Since the Nusselt number is a normalized surface temperature gradient, its functional dependence *for a prescribed geometry* is**

$$Nu = f\left(x^*, \mathrm{Re}, \mathrm{Pr}\right)$$

- **Recalling that the average convection coefficient results from an integration over the entire surface, the *x** dependence disappears and we have:**

$$\overline{Nu} = \frac{\bar{h}L}{k} = f\left(\mathrm{Re}, \mathrm{Pr}\right)$$

Flat Plate in Parallel, Laminar Flow (the Blasius Solution)

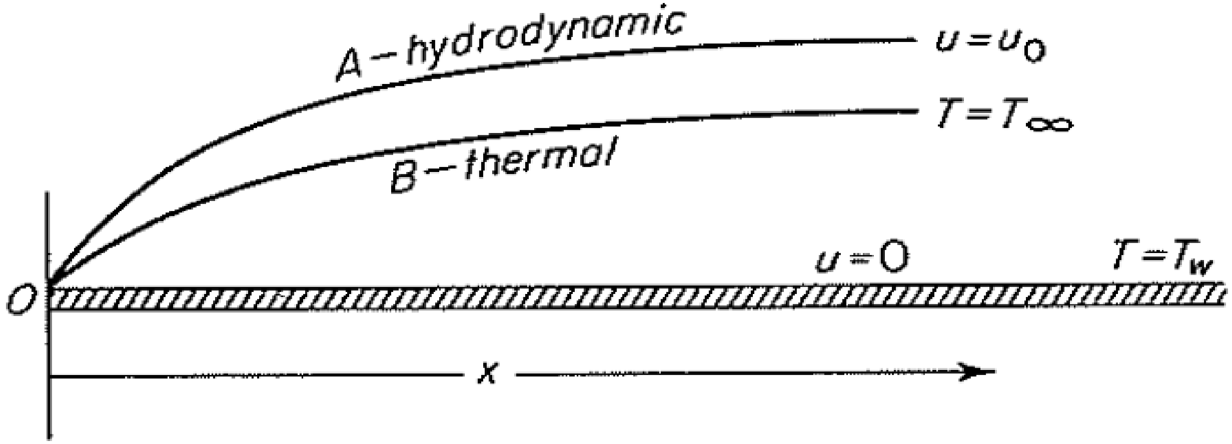
- Assumptions:
  - steady, incompressible flow
  - constant fluid properties
  - negligible viscous dissipation
  - zero pressure gradient (*dp/dx* = 0)

- Governing equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(continuity)}$$

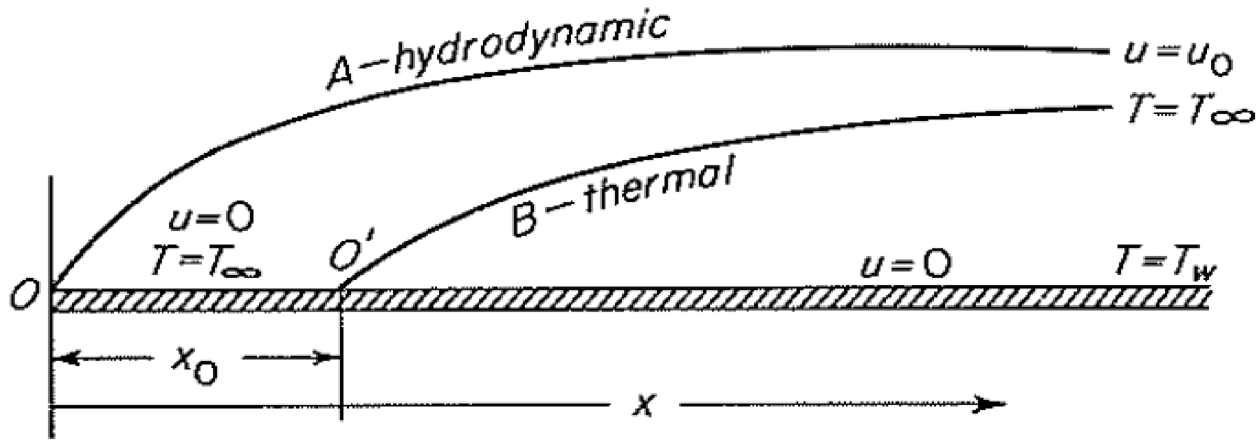
$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \nu\,\frac{\partial^2 u}{\partial y^2} \qquad \text{(momentum)}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha\,\frac{\partial^2 T}{\partial y^2} \qquad \text{(energy)}$$



A—hydrodynamic
u=u0
T=T∞
B—thermal
u=0
T=Tw
O
x

(a)



A—hydrodynamic
u=u0
T=T∞
B—thermal
u=0
T=T∞
O'
u=0
T=Tw
O
x0
x

(b)

Temperature of the plate: from x=0 to x_0 T = T_∞ and for x > x_0 T = T_w

density ρ, conductivity k, specific heat c_p, and viscosity μ.

are constant

$$\left(\frac{dT}{dy}\right)_w = \frac{0.332(T_w - T_\infty)}{\sqrt[3]{1-(x_0/x)^{3/4}}}\sqrt[3]{\frac{c_p\mu}{k}}\sqrt{\frac{u_0\rho}{\mu x}}$$

$$h_x = \frac{k}{T_w - T_\infty}\left(\frac{dT}{dy}\right)_w$$

$$h_x = \frac{0.332k}{\sqrt[3]{1-(x_0/x)^{3/4}}}\sqrt[3]{\frac{c_p\mu}{k}}\sqrt{\frac{u_0\rho}{\mu x}}$$

$$\frac{h_x x}{k} = \frac{0.332}{\sqrt[3]{1-(x_0/x)^{3/4}}} \sqrt[3]{\frac{c_p \mu}{k}} \sqrt{\frac{u_0 x \rho}{\mu}}$$

$$N_{\mathrm{Nu},x} = \frac{0.332}{\sqrt[3]{1-(x_0/x)^{3/4}}} \sqrt[3]{N_{\mathrm{Pr}}} \sqrt{N_{\mathrm{Re},x}}$$

$$N_{\mathrm{Nu},x} = \frac{h_x x}{k} = \frac{k}{y}\frac{x}{k} = \frac{x}{y}$$

For $x_0 = 0$

$$N_{\mathrm{Nu},x} = 0.332 \sqrt[3]{N_{\mathrm{Pr}}} \sqrt{N_{\mathrm{Re},x}}$$

$$N_{Nu} = \frac{hx_1}{k}$$

$$h = \frac{1}{x_1}\int_0^{x_1} h_x\,dx$$

$$h_x = \frac{C}{\sqrt{x}}$$

$$h = \frac{C}{x_1}\int_0^{x_1}\frac{dx}{\sqrt{x}} = \frac{2C}{x_1}\sqrt{x_1} = \frac{2C}{\sqrt{x_1}} = 2h_{x_1}$$

$$N_{Nu} = 0.664\sqrt[3]{N_{Pr}}\sqrt{N_{Re,x_1}}$$

Blasius Solution, cont.

- Blasius derived:

$$\delta = \frac{5.0}{\sqrt{u_\infty / \nu x}} = \frac{5x}{\sqrt{\mathrm{Re}_x}}$$

$$C_{f,x} = 0.664\,\mathrm{Re}_x^{-1/2}$$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\frac{\delta}{\delta_t} \approx \mathrm{Pr}^{1/3}$$

PARALLEL FLOW OVER FLAT PLATES

$$\text{Re}_{cr} = \frac{\rho \upsilon \; x_{cr}}{\mu} = 5\times10^5$$

$$Nu = \frac{hL}{k} = 0.664 \;\; \text{Re}_L^{0.5}\, \text{Pr}^{1/3}$$

$\text{Re}_L < 5\times10^5$ laminar

$$Nu = \frac{hL}{k} = 0.037 \;\; \text{Re}_L^{0.8}\, \text{Pr}^{1/3}$$

$0.6 \le \text{Pr} \le 60$ turbulent

$5\times10^5 \le \text{Re}_L \le 10^7$

Forced Convection Heat Transfer – Internal Flow

Review of Internal (Pipe) Flow Fluid Mechanics

- Flow characteristics
- Reynolds number
- Laminar vs. turbulent flow
- Mean velocity
- Hydrodynamic entry region
- Fully-developed conditions
- Velocity profiles
- Friction factor and pressure drop

Pipe Friction Factor

- **Darcy friction factor, *f*, is a dimensionless parameter related to the pressure drop:**

$$f = \frac{-(dp/dx)D}{\rho u_m^2/2}$$

- **For laminar flow in smooth pipes,**

$$f = 64/\mathrm{Re}_D$$

- **For turbulent flow in smooth pipes,**

$$f = [0.790\ln(\mathrm{Re}_D) - 1.64]^{-2}$$

  – good for $3000 < Re_D < 5 \times 10^6$

Fully-Developed Thermal Conditions

- **While the fluid temperature *T(x,r)* and mean temperature $T_m(x)$ never reach constant values in internal flows, a dimensionless temperature difference does - and this is used to define the fully-developed condition:**

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0$$

- **the following must also be true:**

$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right) = \frac{-\partial T/\partial r}{T_s - T_m} \neq f(x)$$

- **therefore, from the defining equation for *h*:**

$$h = \frac{-k\left(\partial T/\partial r\right)_{r=0}}{T_s - T_m} \neq f(x)$$

i.e., *h* = constant in the fully-developed region

Review of Energy Balance Results

- For <u>any</u> incompressible fluid or ideal gas pipe flow,

$$q_{conv} = \dot{m} c_p (T_{mo} - T_{mi})$$

1) For constant surface heat flux,

$$T_{mo} = T_{mi} + \frac{q_s'' P L}{\dot{m} c_p}$$

2) For constant surface temperature,

$$\frac{T_s - T_{mo}}{T_s - T_{mi}} = \exp\left(-\frac{P L \bar{h}}{\dot{m} c_p} \right)$$

3) For constant ambient fluid temperature,

$$\frac{T_\infty - T_{mo}}{T_\infty - T_{mi}} = \exp\left(-\frac{1}{\dot{m} c_p R_{t,tot}} \right)$$

Convection Correlations for Laminar Flow in Circular Pipes

- Fully-developed conditions with Pr > 0.6:

$$Nu_D = \frac{hD}{k} = 4.36 \qquad (q_s'' = \text{constant})$$
$$= 3.66 \qquad (T_s = \text{constant})$$

  - Note that these correlations are independent of Reynolds number !
  - All properties evaluated at $(T_{mi}+T_{mo})/2$

- Entry region with T_s = const and thermal entry length only **(i.e., Pr >> 1 or unheated starting length):**

$$Nu_D = 3.66 + \frac{0.0668(D/L)\,\mathrm{Re}_D\,\mathrm{Pr}}{1+0.04\left[(D/L)\,\mathrm{Re}_D\,\mathrm{Pr}\right]^{2/3}}$$

Convection Correlations for Turbulent Flow in Circular Pipes

- Fully-developed conditions, smooth wall, q''_s = const or T_s = const

  - Dittus-Boelter equation - fully turbulent flow only (Re_D > 10,000) and 0.7 < Pr < 160

$$Nu_D = 0.023\mathrm{Re}^{4/5}\,\mathrm{Pr}^n$$

where

$$n = 0.3 \ \text{for} \ T_s < T_m \quad \text{(fluid cooling)}$$
$$n = 0.4 \ \text{for} \ T_s > T_m \quad \text{(fluid heating)}$$

Convection Correlations for Turbulent Flow in Circular Pipes, cont.

– Sieder-Tate equation - fully tubulent flow ($Re_D > 10{,}000$),very wide range of Pr (0.7 - 16,700), large property variations

$$Nu_D = 0.027 \mathrm{Re}^{4/5} \mathrm{Pr}^{1/3} (\mu/\mu_s)^{0.14}$$

– Gnielinski equation - transitional and fully-turbulent flow ($3000 < Re_D < 5 \times 10^6$), wide range of Pr (0.5 - 2000)

$$Nu_D = \frac{(f/8)(\mathrm{Re}_D - 1000)\,\mathrm{Pr}}{1 + 12.7 (f/8)^{1/2} (\mathrm{Pr}^{2/3} - 1)}$$

Convection Correlations for Turbulent Flow in Circular Pipes, cont.

- Entry Region
  - Recall that the thermal entry region for turbulent flow is relatively short, I.e.,only 10 to 60*D*
  - Thus, fully-developed correlations are generally valid if $L/D > 60$
  - For $20 < L/D < 60$, Molki & Sparrow suggest

$$\frac{\overline{Nu_D}}{Nu_{D,fd}} = 1 + \frac{6}{L/D}$$

The Reynolds Analogy

- **The BL equations for momentum and energy are similar mathematically and indicate analogous behavior for the transport of momentum and heat**
- **This analogy allows one to determine thermal parameters from velocity parameters and vice-versa (e.g., h-values can be found from viscous drag values)**
- **The heat-momentum analogy is applicable in BLs when *dp*/dx**** $\approx$ **0 (turbulent flow is less sensitive to this)**
- **The modified Reynolds analogy states**

$$Nu = \frac{1}{2} C_f \, \mathrm{Re} \, \mathrm{Pr}^{1/3} \quad \text{for} \quad 0.6 < \mathrm{Pr} < 60$$

  - where C_f is the skin friction coefficient

Colburn Analogy (0.6 < Pr < 100):

$$\left(\frac{h}{C_p G}\right)\left(\frac{C_p \mu}{k}\right)^{2/3}\left(\frac{\mu_w}{\mu}\right)^{0.14} = 0.023\left(\frac{DG}{\mu}\right)^{-0.2}$$

$$j_H = 0.023\left(\frac{DG}{\mu}\right)^{-0.2} = \frac{f}{2} \quad \text{for pipe flow}$$

where j_H is the j-factor for heat transfer, f is the friction factor

$$\left(\frac{h}{C_p G}\right) = \text{Stanton Number} \quad (N_{St})$$

NATURAL CONVECTION

Events due to natural convection

- Weather events such as a thunderstorm
- Glider planes
- Radiator heaters
- Hot air balloon

- Heat flow through and on outside of a double pane window
- Oceanic and atmospheric motions
- Coffee cup example ….

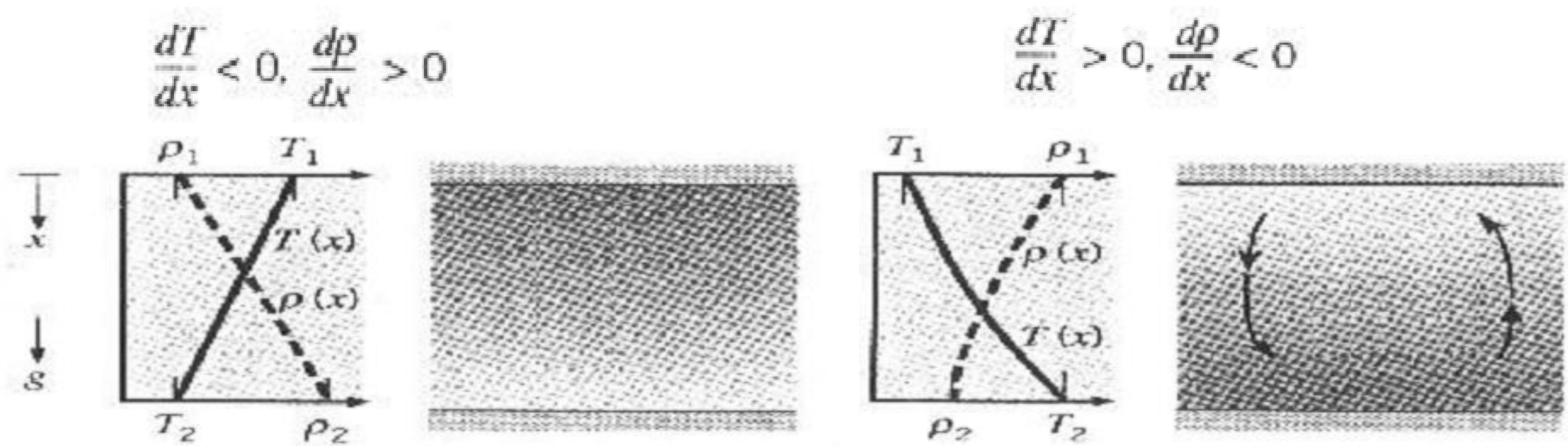
Small velocity

Natural Convection

- New terms
  - ***Volumetric thermal expansion coefficient***
  - ***Grashof number***
  - ***Rayleigh number***
- Buoyancy is the driving force
  - Stable versus unstable conditions
- Nusselt number relationship for laminar free convection on hot or cold surface
- Boundary layer impacts: laminar $\Rightarrow$ turbulent

Buoyancy is the driving force in Natural Convection

- ***Buoyancy*** is due to combination of
  - Differences in fluid density
  - Body force proportional to density
  - Body forces namely, gravity, also Coriolis force in atmosphere and oceans
- Convection flow is driven by buoyancy in unstable conditions
- Fluid motion may be
  (no constraining surface) or along a surface



Governing Equations

- Define β, the volumetric thermal expansion coefficient,

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \quad = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

For all liquids and gases

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T}$$

$$\text{For an ideal gas : } P = \frac{RT}{\rho} \Rightarrow \rho = \frac{P}{RT}$$

$$Thus : \beta = \frac{1}{T}$$

$$\rho_\infty - \rho \approx \rho\beta(T - T_\infty)$$

Density gradient is due to the temperature gradient

Dimensionless Similarity Parameter

- Define new dimensionless parameter,

$$Gr_L = \frac{g\beta(T_s - T_\infty)L}{u_0^2}\left(\frac{u_0 L}{\nu}\right)^2 = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

•***Grashof number*** in natural convection is analogous to the Reynolds number in forced convection.

•***Grashof number*** indicates the ratio of the buoyancy force to the viscous force.

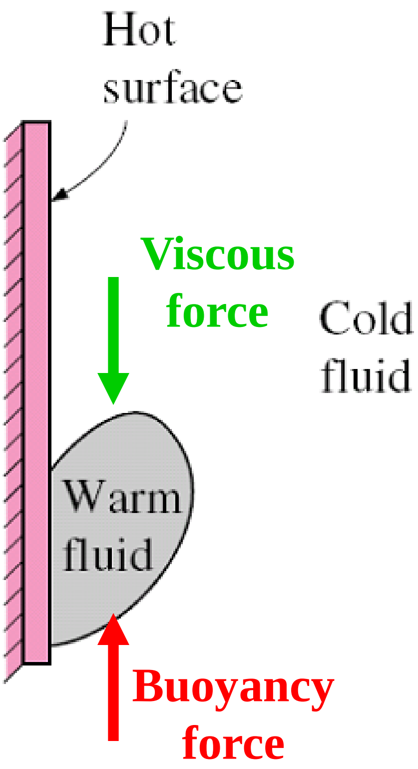
•Higher Gr number means increased natural convection flow

$$\frac{Gr_L}{\mathrm{Re}_L^2} << 1 \quad \text{forced} \qquad\qquad \frac{Gr_L}{\mathrm{Re}_L^2} >> 1 \quad \text{natural}$$

- The dimensionless parameter **Grashof number** Gr_L

$$Gr_L = \frac{g\beta\left(T_s - T_\infty\right)L_c^3}{\nu^2}$$

$$Gr_L = \frac{\textit{Buoyancy force}}{\textit{Viscous force}}$$



- The flow regime in natural convection governed by the *Grashof number*

$Gr_L > 10^9$ flow is turbulent

Empirical Correlations

Typical correlations for heat transfer coefficient developed from experimental data are expressed as:

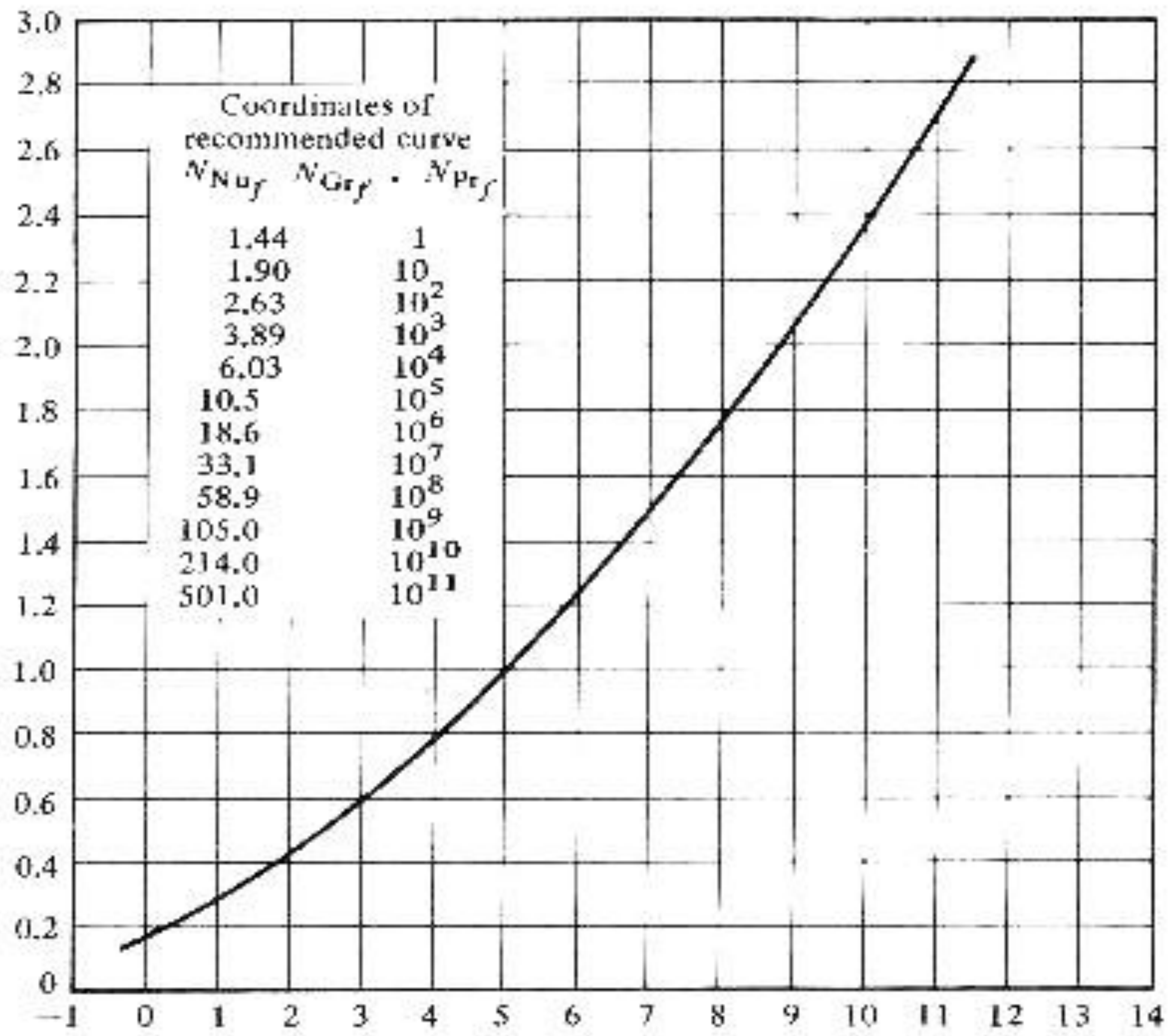
$$\overline{Nu_L} = \frac{\bar{h}L}{k} = CRa_L^n$$

$$Ra_L = Gr_L \cdot \Pr = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha}$$

$$\begin{cases} n = 1/4 & \text{For Turbulent} \\ n = 1/3 & \text{For Laminar} \end{cases}$$

Vertical Plate at constant *Ts*

$Log_{10} Nu_L$

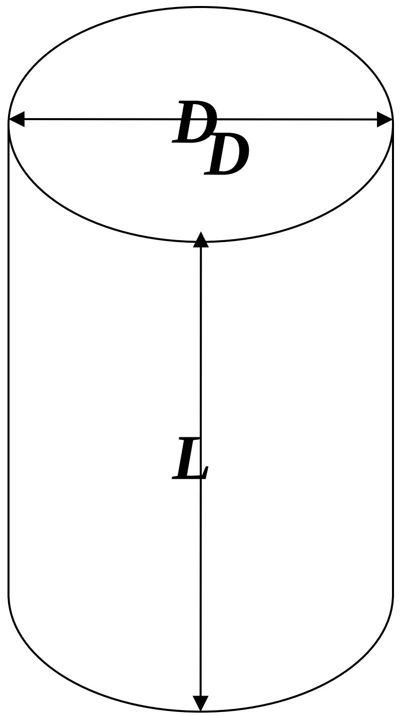


$Log_{10} Ra_L$

•Alternative applicable to entire Rayleigh number range (for constant *Ts*)

$$\overline{Nu_L} = \left\{ 0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1 + (0.492/\mathrm{Pr})^{9/16} \right]^{8/27}} \right\}^2$$

Vertical Cylinders

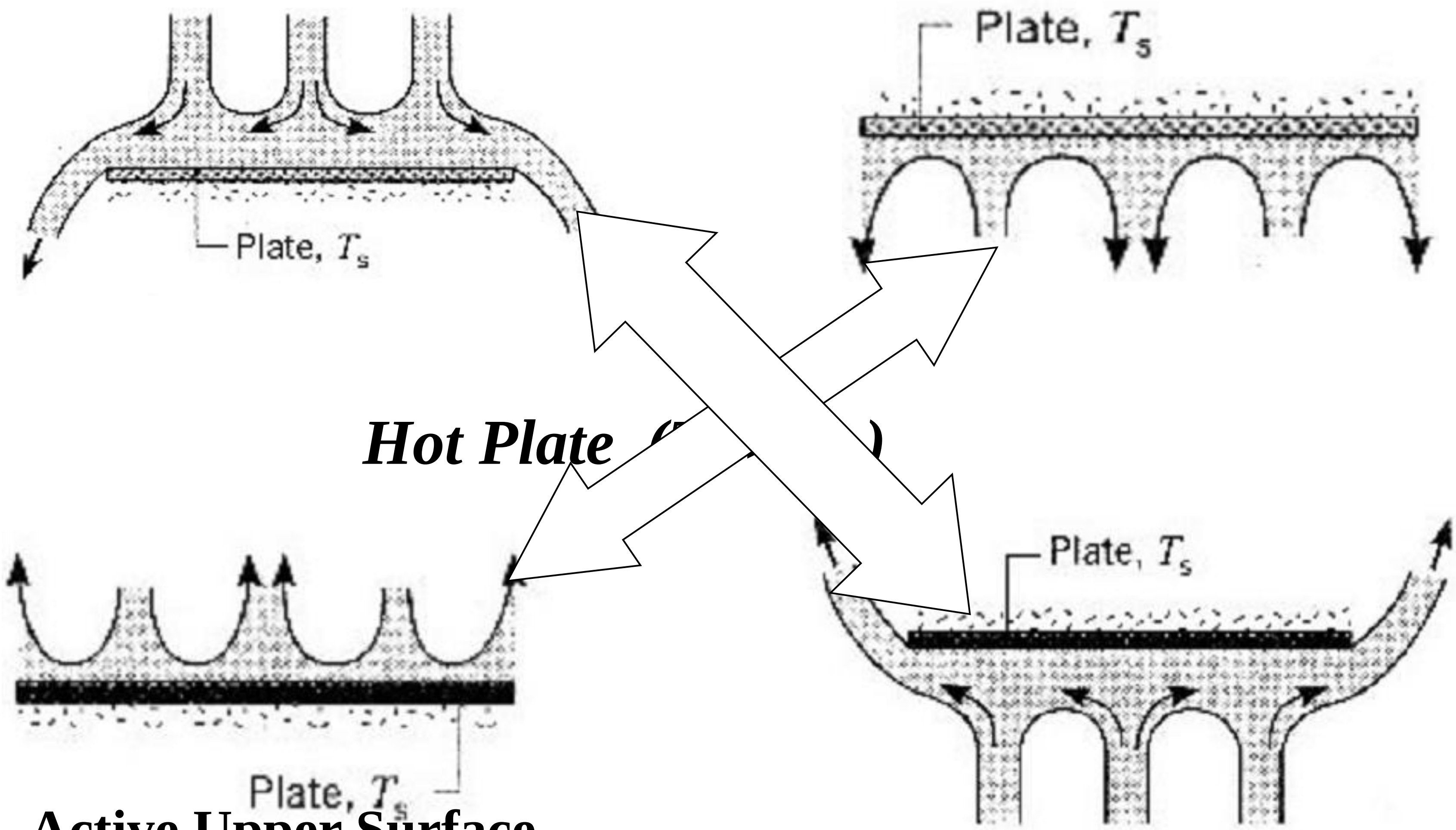


•Use same correlations for vertical flat plate if:

$$\frac{D}{L} \gtrsim \frac{35}{Gr_L^{1/4}}$$

Horizontal Plate

Cold Plate ($T_s < T_\infty$)



Hot Plate

Active Upper Surface

Active Lower Surface

Empirical Correlations : **Horizontal Plate**

- Define the characteristic length, L as

$$L \equiv \frac{A_s}{P}$$

- **Upper surface of heated plate, or Lower surface of cooled plate** :

$$\overline{Nu_L} = 0.54\,Ra_L^{1/4} \qquad \left(10^4 \le Ra_L \le 10^7\right)$$

$$\overline{Nu_L} = 0.15\,Ra_L^{1/3} \qquad \left(10^7 \le Ra_L \le 10^{11}\right)$$

- **Lower surface of heated plate, or Upper surface of cooled plate** :

$$\overline{Nu_L} = 0.27\,Ra_L^{1/4} \qquad \left(10^5 \le Ra_L \le 10^{10}\right)$$

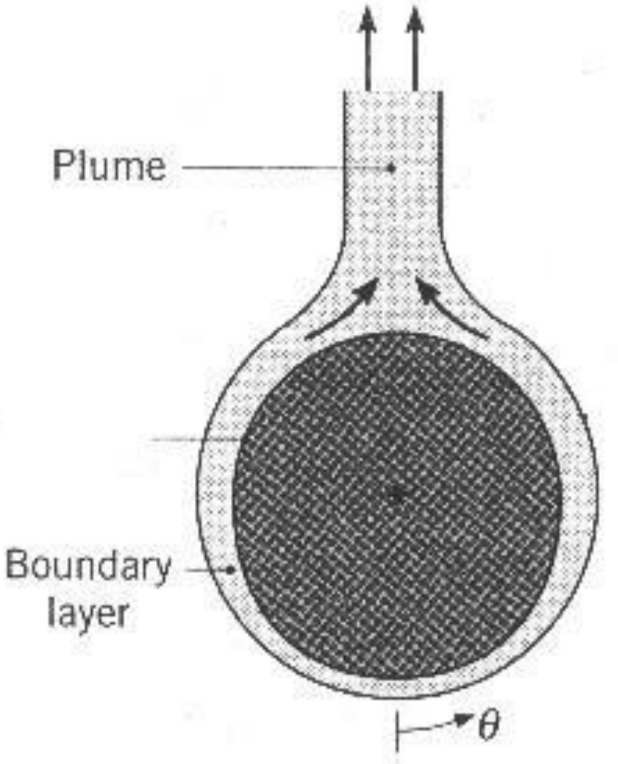
Note: Use fluid properties at the ***film temperature***

$$T_f = \frac{T_s + T_\infty}{2}$$

Empirical Correlations : **Long Horizontal Cylinder**

- Very common geometry (pipes, wires)
- For isothermal cylinder surface, use general form equation for computing Nusselt #

$$\overline{Nu_D} = \frac{\bar{h} D}{k} = C Ra_D^n$$



Constants for general Nusselt number Equation

Ra_D	C	n
10^{-10} - 10^{-2}	0.675	0.058
10^{-2} - 10^{+2}	1.02	0.148
10^{2} - 10^{4}	0.850	0.188
10^{4} - 10^{7}	0.480	0.250
10^{7} - 10^{12}	0.125	0.333

Nusselt Number Correlations for Enclosures

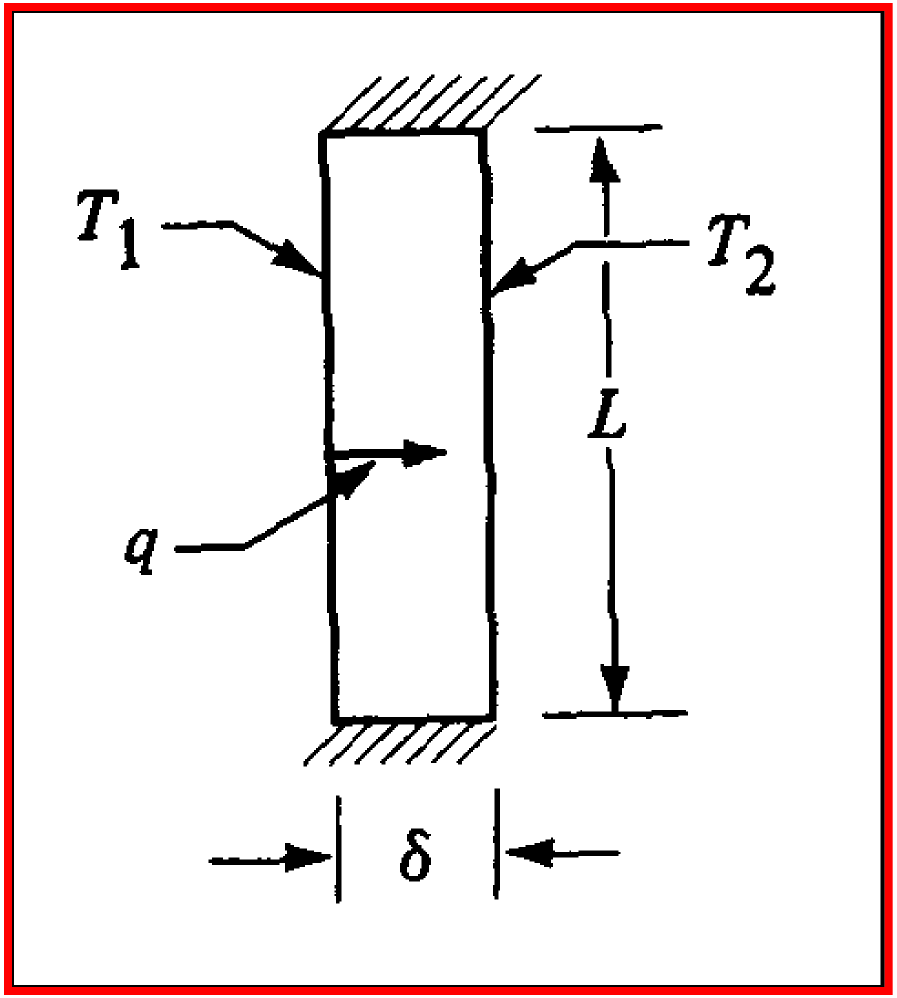
- Simple power-law type relations in the form of

$$Nu = C \cdot Ra_L^n$$

where *C* and *n* are constants, are sufficiently accurate, but they are usually applicable to a narrow range of Prandtl and Rayleigh numbers and aspect ratios.

- Numerous correlations are widely available for
  - horizontal rectangular enclosures,
  - inclined rectangular enclosures,
  - vertical rectangular enclosures,
  - concentric cylinders,
  - concentric spheres.

Natural Convection in Enclosed Spaces



Two vertical plates separated by a distance. Each plate at a different temperature.

Ends are insulated.

Convective heat transfer occurs in the fluid within the space.

$$N_{Gr,\delta} = \frac{\delta^3 \rho^2 g \beta \left(T_1 - T_2\right)}{\mu^2}$$

$$N_{Nu,\delta} = \frac{h\delta}{k}$$

$$\frac{q}{A} = h\left(T_1 - T_2\right)$$

Natural Convection in Enclosed Spaces

For gases enclosed between vertical plates,

$$N_{Nu,\delta} = \frac{h\delta}{k} = 1.0 \qquad \text{for } N_{Gr,\delta} N_{\text{Pr}} < 2 \times 10^3$$

$$N_{Nu,\delta} = 0.20 \frac{\left(N_{Gr,\delta} N_{\text{Pr}}\right)^{1/4}}{\left(L/\delta\right)^{1/9}} \qquad \text{for } 6 \times 10^3 < N_{Gr,\delta} N_{\text{Pr}} < 2 \times 10^5$$

$$N_{Nu,\delta} = 0.0073 \frac{\left(N_{Gr,\delta} N_{\text{Pr}}\right)^{1/3}}{\left(L/\delta\right)^{1/9}} \qquad \text{for } 2 \times 10^5 < N_{Gr,\delta} N_{\text{Pr}} < 2 \times 10^7$$

For liquids enclosed between vertical plates,

$$N_{Nu,\delta} = \frac{h\delta}{k} = 1.0 \qquad \text{for } N_{Gr,\delta} N_{\text{Pr}} < 1 \times 10^3$$

$$N_{Nu,\delta} = 0.28 \frac{\left(N_{Gr,\delta} N_{\text{Pr}}\right)^{1/4}}{\left(L/\delta\right)^{1/4}} \qquad \text{for } 1 \times 10^3 < N_{Gr,\delta} N_{\text{Pr}} < 1 \times 10^7$$

Natural Convection in Enclosed Spaces

For gases enclosed between horizontal plates,
with lower plate hotter than upper

$$N_{Nu,\delta} = 0.20\left(N_{Gr,\delta} N_{\text{Pr}}\right)^{1/4} \quad \text{for } 7\times10^{3} < N_{Gr,\delta} N_{\text{Pr}} < 3\times10^{5}$$

$$N_{Nu,\delta} = 0.061\left(N_{Gr,\delta} N_{\text{Pr}}\right)^{1/3} \quad \text{for } 3\times10^{5} < N_{Gr,\delta} N_{\text{Pr}}$$

For liquids enclosed between vertical plates,
with lower plate hotter

$$N_{Nu,\delta} = 0.069\left(N_{Gr,\delta} N_{\text{Pr}}\right)^{1/3} N_{\text{Pr}}^{0.074} \quad \text{for } 1.5\times10^{5} < N_{Gr,\delta} N_{\text{Pr}} < 1\times10^{9}$$

Combined Natural and Forced Convection

- Heat transfer coefficients in forced convection are typically much higher than in natural convection.
- The error involved in ignoring natural convection may be considerable at low velocities.
- Nusselt Number:
  - Forced convection (flat plate, laminar flow):

$$Nu_{\text{forced convection}} \propto \mathrm{Re}^{1/2}$$

  - Natural convection (vertical plate, laminar flow):

$$Nu_{\text{natural convection}} \propto Gr^{1/4}$$

- Therefore, the parameter Gr/Re^2 represents the importance of natural convection relative to forced convection.

Previous GATE Questions

SOLVED PROBLEMS

Q1:

In forced convection, the Nusselt number Nu is a function of

A. Re and Pr

B. Re and Gr

C. Pr and Gr

D. Re and Sc

Q2:

The unit of resistance to heat transfer is

A. J/m^2-K

B. J/m-K

C. W/m^2-Kh

D. m^2-K/W

Q3:

A fluid is flowing inside the inner tube of a double pipe heat exchanger with diameter 'D'. For a fixed mass flow rate, the tube side heat transfer coefficient for turbulent flow conditions is proportional to

The Sieder-Tate correlation for heat transfer in turbulent from in a pipe gives Nu α $Re^{0.8}$, where Nu is the Nusselt number and Re is the Reynolds number for the flow. Assuming that this relation is valid, the heat transfer coefficient varies with pipe diameter D as

A. $D^{0.8}$

B. $D^{-0.2}$

C. D^{-1}

D. $D^{-1.8}$

$$Nu \alpha Re^{0.8} \Rightarrow \frac{hD}{K} \alpha \left(\frac{V\rho D}{\mu}\right)^{0.8}$$

$$\Rightarrow \frac{hD}{K} \alpha \frac{V^{0.8}\rho^{0.8}D^{0.8}}{\mu^{0.8}} \Rightarrow h \alpha V^{0.8} \rho^{0.8} D^{-0.2} K$$

$\therefore$ For constant average velocity, $h \alpha D^{-0.2}$

Q4:

Heat transfer by natural convection is enhanced in systems with

A. high viscosity
B. **high coefficient of thermal expansion**
C. low temperature gradients
D. low density change with temperature

Q5:

For turbulent flow in a tube, the heat transfer coefficient is obtained from the Dittus-Boelter equation. If the tube diameter is halved and the flow rate is doubled, then the heat transfer coefficient will change by a factor of

A. 1
B. 1.74
C. 6.1
D. 37

$$Nu = \frac{hD}{k} = 0.023\, Re^{0.8}\, pr^{n}$$

$$Re = \frac{DV\rho}{\mu}$$

D is halved $\frac{D_1}{D_2} = 2$

Q is doubled. $\frac{Q_1}{Q_2} = \frac{1}{2}$

$$Q = \left(\frac{\pi D^2}{4}\right) \cdot v$$

= Area . velocity.

$$\Rightarrow v = \frac{4Q}{\pi D^2}$$

$$Re = \frac{D\rho}{\mu} \cdot \frac{4Q}{\pi D^2} = \frac{4\rho}{\pi\mu}\left(\frac{Q}{D}\right).$$

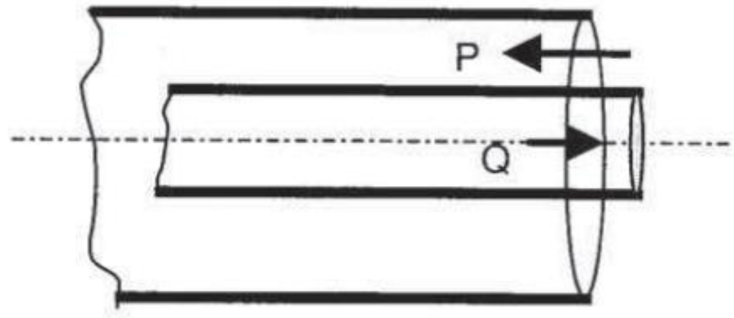
$$\frac{Re_2}{Re_1} = \frac{Q_2}{Q_1} \cdot \frac{D_1}{D_2} = 2 \times 2 = 4.$$

$$h = (0.023\, pr.k)\left(\frac{Re^{0.8}}{D}\right).$$

$$\frac{h_2}{h_1} = \left(\frac{Re_2}{Re_1}\right)^{0.8}\left(\frac{D_1}{D_2}\right) = 4^{0.8}(2) = 6.1$$

Q6:

Two liquids (P and Q) having same viscosity are flowing through a double pipe heat exchanger as shown in the schematic below.



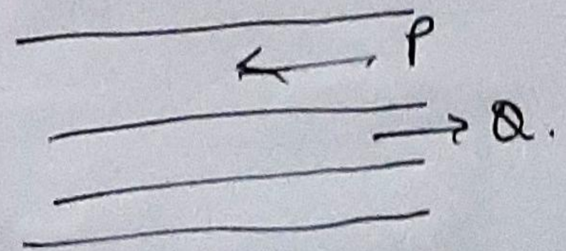
Densities of P and Q are 1000 and 800 kg/m^3 respectively. The average velocities of the liquids P and Q are 1 and 2.5 m/s respectively. The inner diameters of the pipes are 0.31 and 0.1 m. Both pipes are 5 mm thick. The ratio of the Reynolds numbers Re_P to Re_Q is

A. 2.5
B. 1.55
C. 1
D. 4

$$Re = \frac{DV\rho}{\mu}$$

$$\frac{Re_P}{Re_Q} = \frac{0.21 \times 1 \times 1000}{0.1 \times 2.5 \times 800}$$

$$\approx 1.0$$



P	Q
$\rho = 1000\ kg/m^3$	$800\ kg/m^3$
$V = 1\ m/s$	$2.5\ m/s$
$\mu = const$	
$D_P = 0.31\,m$	$D_Q = 0.1\,m$

$$D_{eq} = 0.31 - 0.1$$
$$= 0.21$$

Q7:

Widely used Dittus-Boelter equation is valid provide:

A. 2100 < Re < 10,000 and the properties of the fluid are evaluated at the average temperature
B. Re < 2100 and the properties of the fluid are evaluated at the bulk temperature
C. 10,000 < Re < 120,000 and the properties of the fluid are evaluated at the bulk temperature
D. None of the above

Answer: C

Q8:

A hot horizontal plate is exposed to air by keeping,

A. the hot surface facing up.
B. the hot surface facing down.

Heat transfer to the ambient air is primarily by natural convection. In which of the above cases, is the heat transfer coefficient higher and why?

Ans: When the hot surface is facing up, it is easier for the natural convection to set up (hot air becomes lighter and rises up and the colder air moves in) compared to the case when the hot surface is facing down. **Therefore, the heat transfer coefficient in case (A) would be higher.**

Q9:

For a laminar flow of fluid in a circular tube, h_i is the convective heat transfer coefficient at a velocity V_1. If the velocity is reduced by half and assuming the fluid properties are constant, the new convective heat transfer coefficient is:

A. $1.26h_i$
B. $0.794h_i$
C. $0.574h_i$
D. $1.741h_i$

- $Nu = 1.86(Re.\ Pr.\ D/L)^{1/3}$
- So $h_i \propto Re^{1/3}$ means $h \propto v^{1/3}$
- Velocity is reduced by ½
- Substituting and converting

$$h_2 = 0.741h_1$$

Q10:

In natural convection heat transfer the correlating parameter is:

A. Graetz number
B. Ecker number
C. Grashof number
D. Bond number

Answer: C

Q11:

The non-dimensional temperature gradient in a liquid at the wall of the pipe is

A. The heat flux
B. Nussult number
C. Prandtl number
D. Schmidt number

Answer: B

- Q12:

Heat Transfer occurs by natural convection because change in temperature causes difference in

A. Viscosity

B. Density

C. Thermal conductivity

D. Heat capacity

Answer: B

Q13:

A fluid flows through a cylindrical pipe under fully developed, steady state laminar flow conditions. The tube wall is maintained at constant temperature. Assuming constant physical properties and negligible viscous heat dissipation, the governing equation for the temperature profile is (z -axial direction; r -radial direction)

$$\text{(A)}\ U_{\max}\left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{\partial T}{\partial z}\right)=\frac{k}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]$$

$$\text{(B)}\ U_{\max}\left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{\partial T}{\partial r}\right)=\frac{k}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial z}\right)+\frac{\partial^2 T}{\partial r^2}\right]$$

$$\text{(C)}\ 2\,U_{\max}\left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{\partial^2 T}{\partial z^2}\right)=\frac{k}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial r^2}\right]$$

$$\text{(D)}\ U_{\max}\left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{\partial T}{\partial z}\right)=\frac{k}{\rho C_p}\left[\frac{1}{z}\frac{\partial}{\partial z}\left(z\frac{\partial T}{\partial z}\right)+\frac{\partial^2 T}{\partial r^2}\right]$$

Answer: A

Q14:

Prandtl number signifies the ratio of

A. Momentum Diffusivity /Thermal Diffusivity
B. Mass Diffusivity/Thermal Diffusivity
C. Thermal Diffusivity/Momentum Diffusivity
D. Thermal Diffusivity/Mass Diffusivity

- Answer: A

Q15:

A fluid flows over a heated horizontal plate maintained at temperature T_w. The bulk temperature of the fluid is T_∞. The temperature profile in the thermal boundary layer is given by:

$$T = T_w + (T_w - T_\infty)\left[\frac{1}{2}\left(\frac{y}{\delta_t}\right)^3 - \frac{3}{2}\left(\frac{y}{\delta_t}\right)\right], \qquad 0 \le y \le \delta_t$$

Here, y is the vertical distance from the plate, δ_t is the thickness of the thermal boundary layer and k is the thermal conductivity of the fluid.

The local heat transfer coefficient is given by

(A) $\dfrac{k}{2\delta_t}$ (B) $\dfrac{k}{\delta_t}$ (C) $\dfrac{3}{2}\dfrac{k}{\delta_t}$ (D) $2\dfrac{k}{\delta_t}$

Local heat transfer co-efficient & heat flux are given by

$$\frac{q_x}{A} = h_x (T_w - T_\infty) = -k \frac{dT}{dy}\bigg|_{y=0}$$

but given

$$T = T_w + (T_w - T_\infty)\left[\frac{1}{2}\left(\frac{y}{\delta_t}\right)^3 - \frac{3}{2}\left(\frac{y}{\delta_t}\right)\right] \quad 0 \le y \le \delta_t$$

$$\Rightarrow \frac{dT}{dy} = (T_w - T_\infty)\left[\frac{1}{2(\delta_t)^3} \cdot (3y^2) - \frac{3}{2\delta_t}\right]$$

$$\frac{dT}{dy}\bigg|_{y=0} = (T_w - T_\infty)\left(-\frac{3}{2}\frac{1}{\delta_t}\right).$$

$$\therefore \quad h (T_w - T_\infty) = -k (T_w - T_\infty) \frac{3}{2} \frac{1}{\delta_t}$$

$$\Rightarrow h = \frac{3}{2} \frac{k}{\delta_t}.$$

Q16:

Match the dimensionless numbers in **Group-1** with the ratios in **Group-2**.

	Group-1		Group-2
P	Biot number	I	$\dfrac{\text{buoyancy force}}{\text{viscous force}}$
Q	Schmidt number	II	$\dfrac{\text{internal thermal resistance of a solid}}{\text{boundary layer thermal resistance}}$
R	Grashof number	III	$\dfrac{\text{momentum diffusivity}}{\text{mass diffusivity}}$

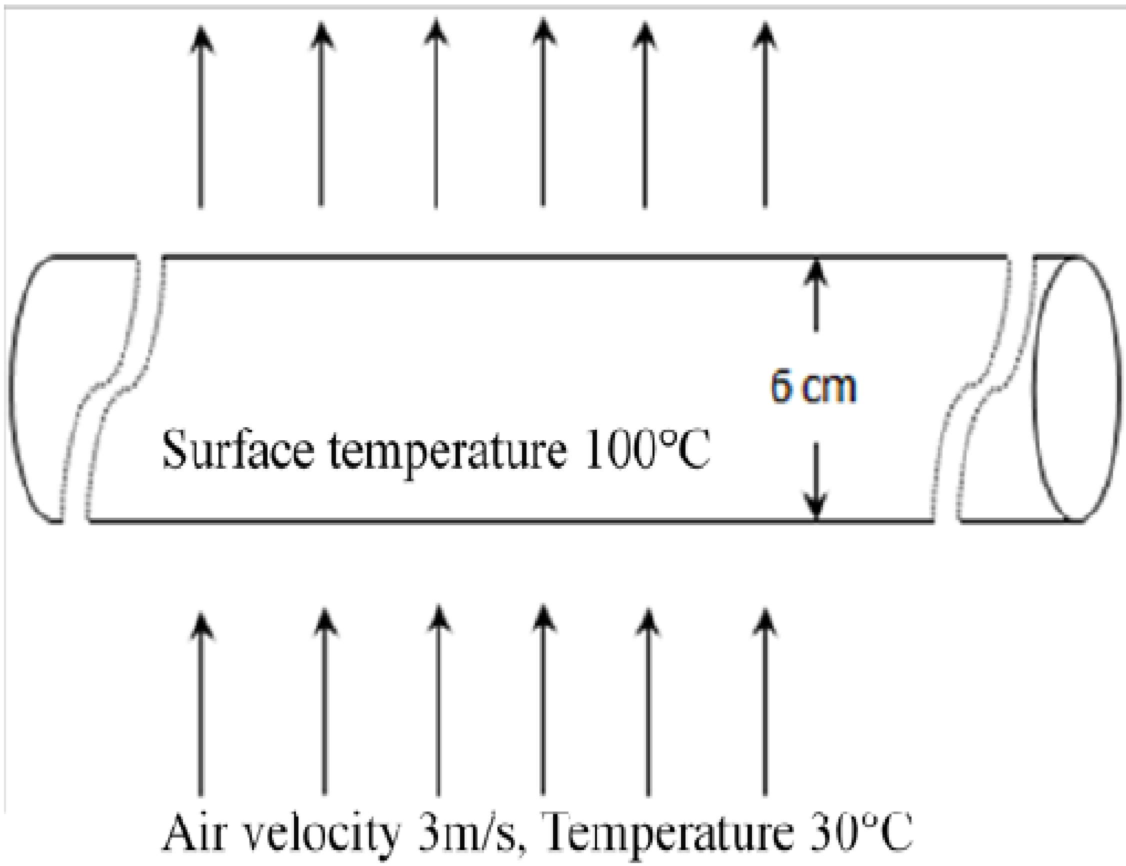
(A) P-II, Q-I, R-III (B) P-I, Q-III, R-II

(C) P-III, Q-I, R-II (D) P-II, Q-III, R-I

Answer: D

Q17:

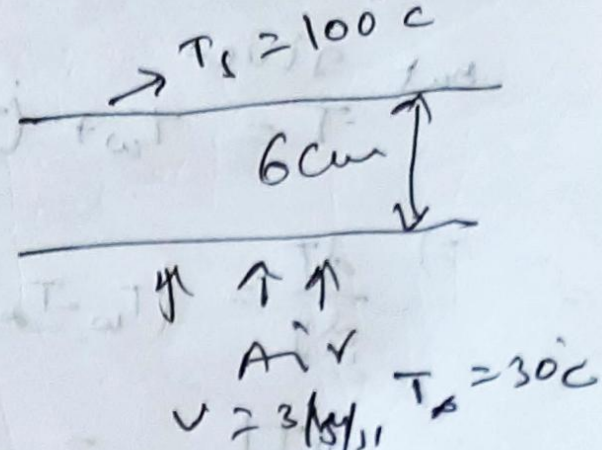
Air is flowing at a velocity of 3 m/s perpendicular to a long pipe as shown in the figure below. The outer diameter of the pipe is d = 6 cm and temperature at the outside surface of the pipe is maintained at 100 °C. The temperature of the air far from the tube is 30 °C. Data for air: Kinematic viscosity, v = 18×10−6 m2/s; Thermal conductivity, k = 0.03 W/(m.K) Using the Nusselt number correlation: $Nu = 0.024Re^{0.8}$, the rate of heat loss per unit length (W/m) from the pipe to air (up to one decimal place) is ___________________.


6 cm
Surface temperature 100°C
Air velocity 3m/s, Temperature 30°C

$$\nu = 18 \times 10^{-6}\ m^2/s$$

$$k = 0.03\ W/mK$$

$$Nu = 0.024 \times Re^{0.8}$$



$$Re = \frac{DV\rho}{\mu} = \frac{DV}{\nu}$$

$$= \frac{0.06 \times 3}{18 \times 10^{-6}} = 10,000$$

$$Nu = 0.024 (40{,}000)^{0.8}$$
$$= 38.037$$

$$Nu = \frac{hD}{k} = 38.037 \Rightarrow h = \frac{Nu \times k}{D}$$

$$\Rightarrow h = 38.037 \times 0.03 \times \frac{1}{0.06}$$
$$= 19.019$$

rate of heat loss per unit length

$$(q/L) = hA(T_s - T_\infty)$$
$$= h\pi D (T_s - T_\infty)$$
$$= 19.019 \times \pi \times 0.06 (100-30)$$
$$= 250.95 \text{ W/m}.$$