GATE

Analog Communication

Dr. Ch.Srinivasa Rao,
Professor of ECE, UCEV
JNTUK, Vizianagaram

Day-1

- Review of Signals
- Review of Fourier series
- Review of Fourier Transform
- Review of Hilbert Transform
- Sample (Old GATE) Problems
- Need for Modulation

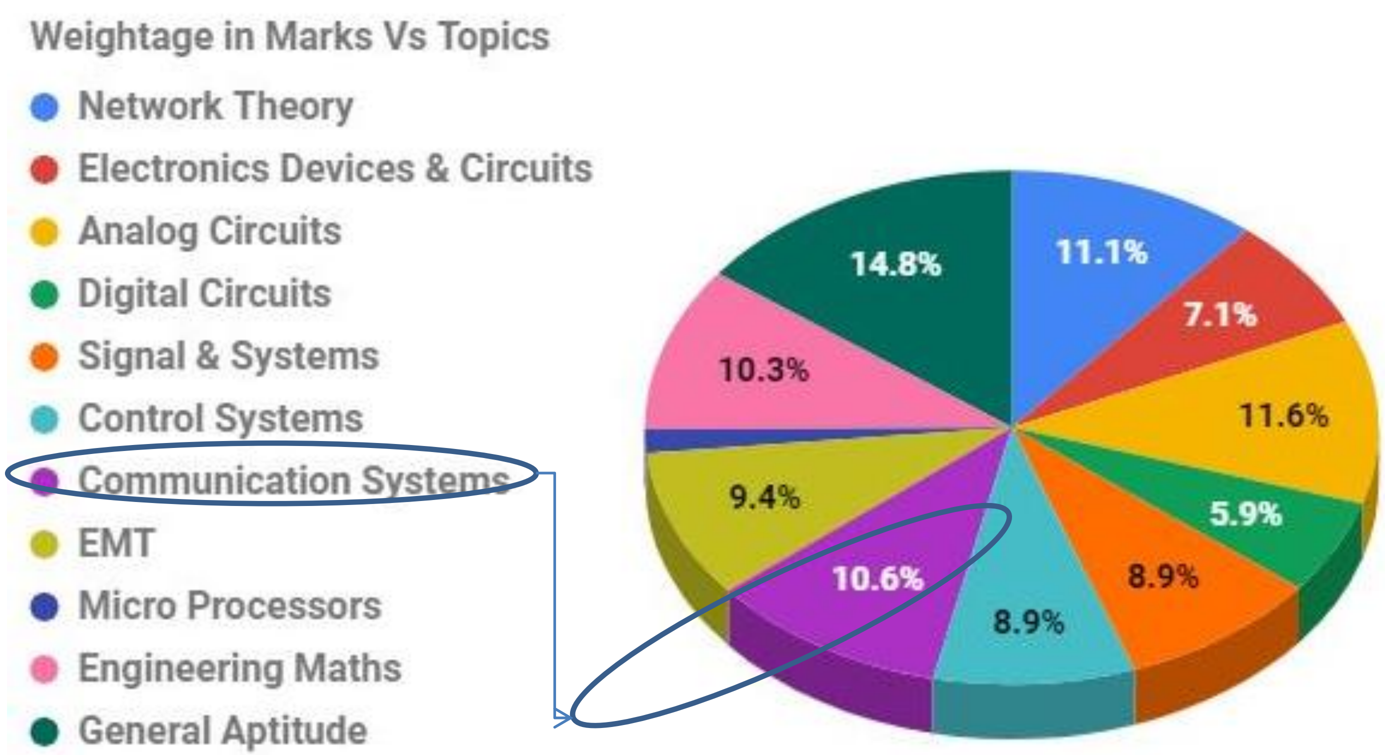
Communications Syllabus

Random processes: Autocorrelation and power spectral density, properties of white noise, filtering of random signals through LTI systems;

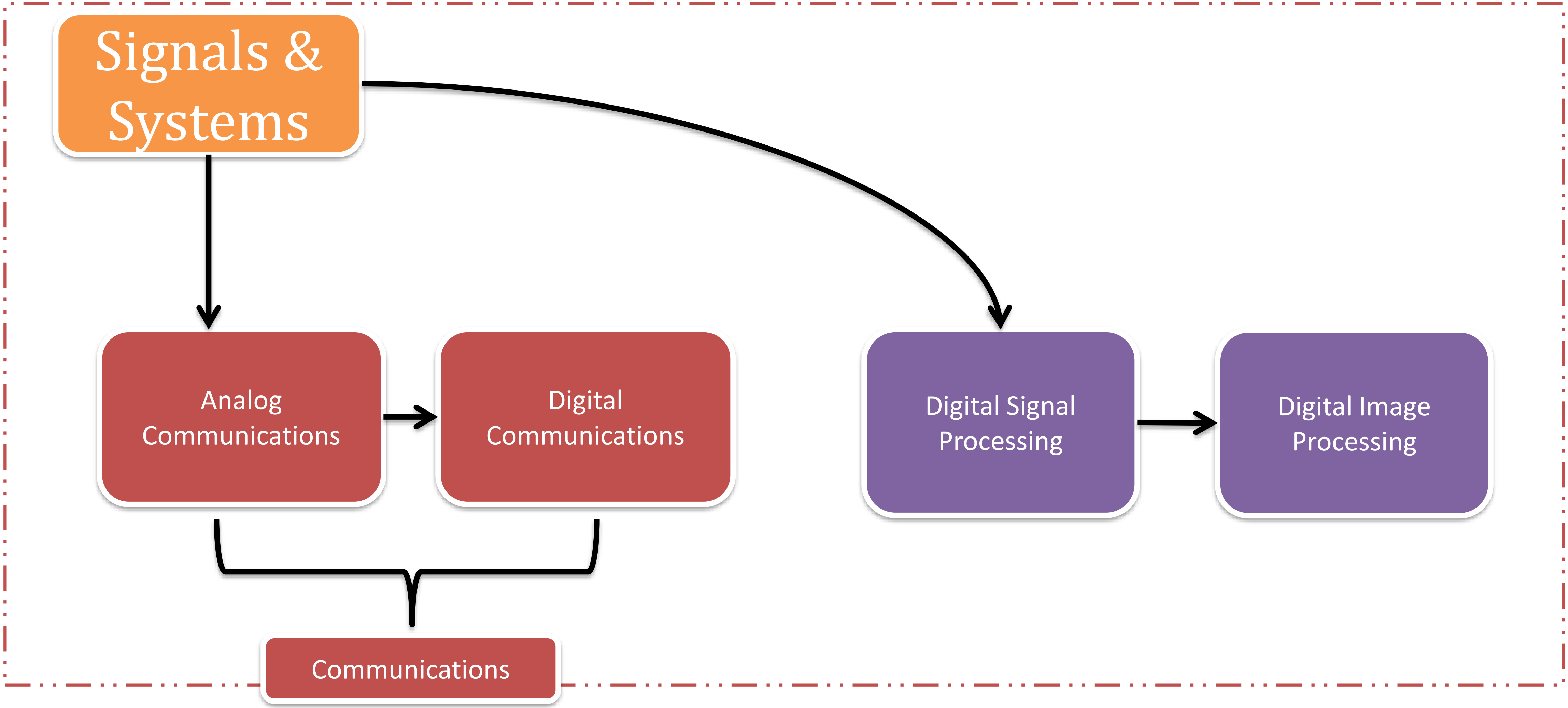
Analog communications: Amplitude modulation and demodulation, angle modulation and demodulation, spectra of AM and FM, superheterodyne receivers, circuits for analog communications; Information theory: entropy, mutual information and channel capacity theorem.

Digital communications: PCM, DPCM, digital modulation schemes, amplitude, phase and frequency shift keying (ASK, PSK, FSK), QAM, MAP and ML decoding, matched filter receiver, calculation of bandwidth, SNR and BER for digital modulation; Fundamentals of error correction, Hamming codes; Timing and frequency synchronization, inter-symbol interference and its mitigation; Basics of TDMA, FDMA and CDMA.

GATE Subject Wise Weightage for ECE 2020



Prerequisite



Analog Communication
Presentation Outline (Session/Day wise)

1. Introduction to Signals, Fourier Analysis and Hilbert Transform
2. CW Modulation- Full carrier AM and problems
3. CW Modulation- Suppressed carrier AM and problems
4. CW Modulation- Angle Modulation (NBFM ,WBFM) and problems
5. CW Modulation- FM , PM Generation and detection with problems
6. AM,FM Transmitters and Receivers
7. AM, FM Receivers and problems
8. Noise
9. Noise in CW Modulation schemes with Problems
10. Sampling
11. Pulse Analog modulation
12. Information Theory and Coding

Presentation Outline- Session/Day 1

- Introduction to Signals
- CT and DT Signals
- Singularity functions
- Fourier Series
- Problems on Fourier Series
- Fourier Transform
- Problems on Fourier Transform
- Hilbert Transform and its Properties
- Electronic communication system
- Need for modulation

Introduction

SIGNAL:

A signal is a function of one or more variables that conveys information about some (usually physical) phenomenon.

(or)

A signal is a physical quantity that varies with time, space or any other independent variable or variables.

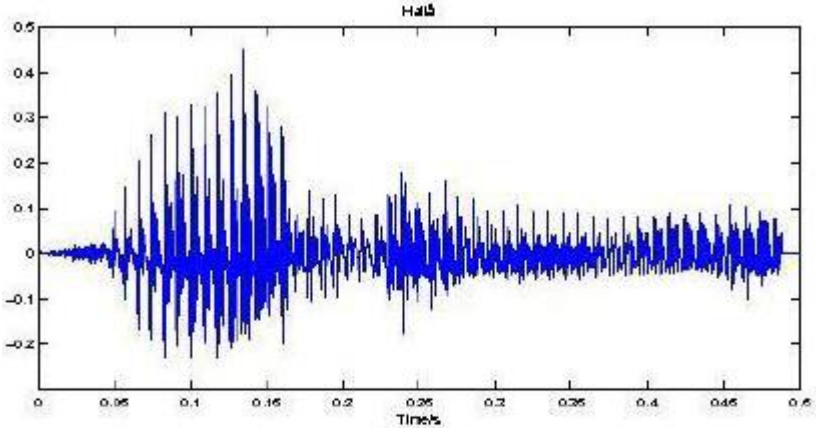
Signals are represented mathematically as functions of one or more independent variables.

Ex: Speech signal, Image, Video

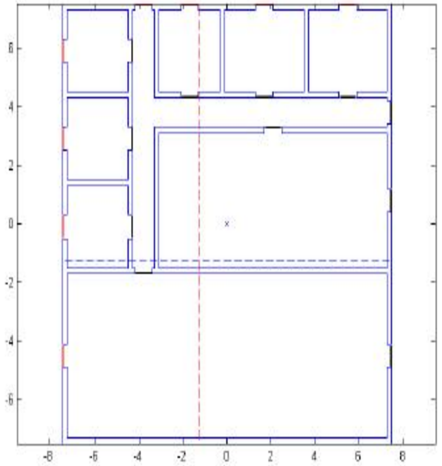
It possesses uncertainty (digital) or randomness (analog) and should have a band width.

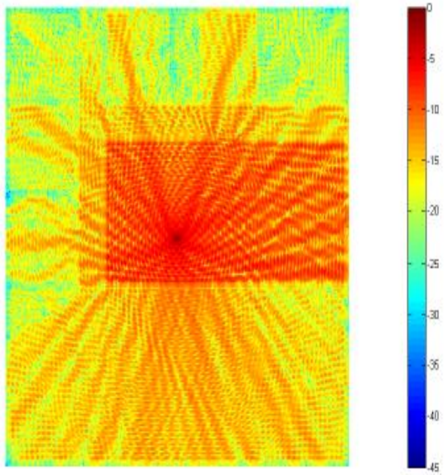
Introduction

Signals vary with **time**
Ex: Speech signal
No functional relationship
to describe the signal

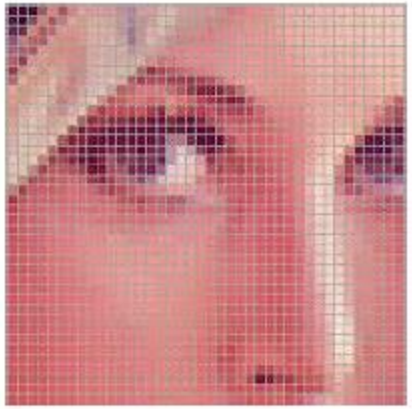


Signals vary with **space**
Ex: Electromagnetic field within a room





Signals vary with **dimensions**
Ex: The RGB value of an image

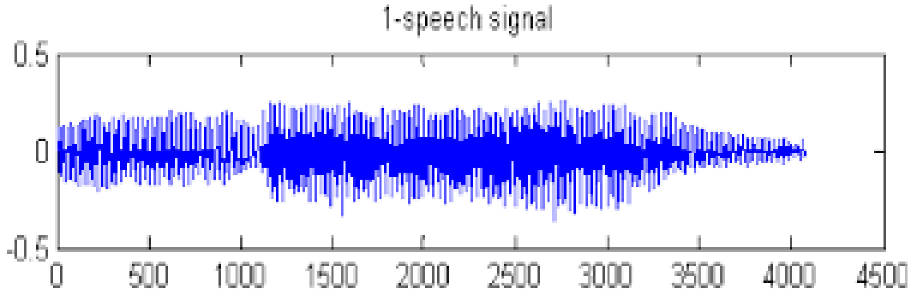


Introduction

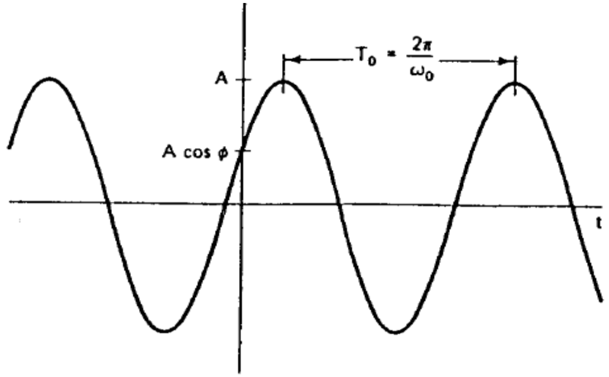
- Signals can be classified based on the number of independent variables with which they are associated.
- If a signal is a function of only one variable, then it is known as one dimensional (1D) signal.

Examples

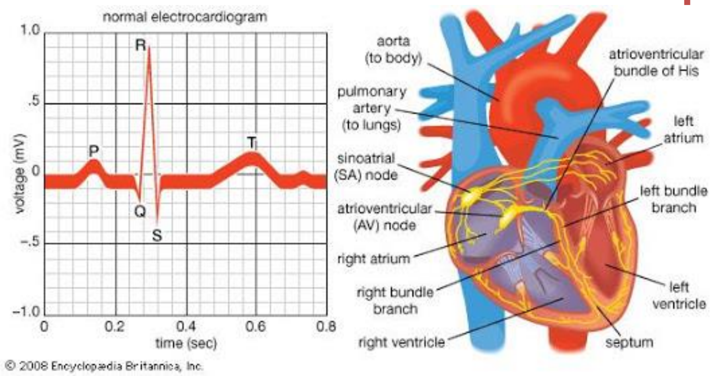
1. Human speech,
2. AC power signal,
3. Electrocardiogram



Speech Signal



AC Power supply



ECG

Introduction

- Similarly, if a signal is a function of two or more variables is said to be multidimensional.. if a signal is a function two variables is said to be two dimensional.
- Examples.

  1.Pictures,

  2.X-Ray images

  3.Sonograms.

s(x,y)

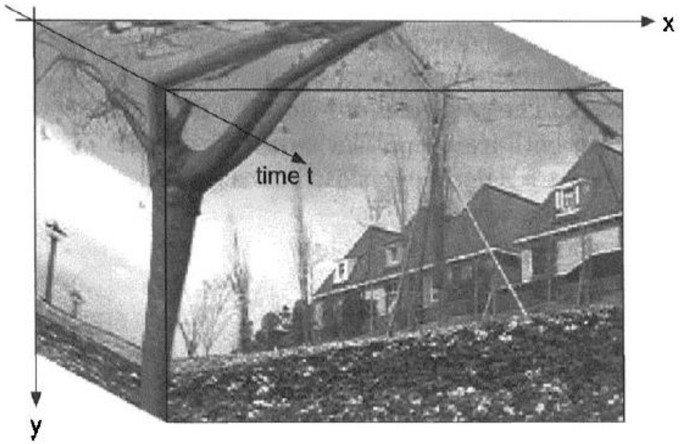
Picture

X-Ray

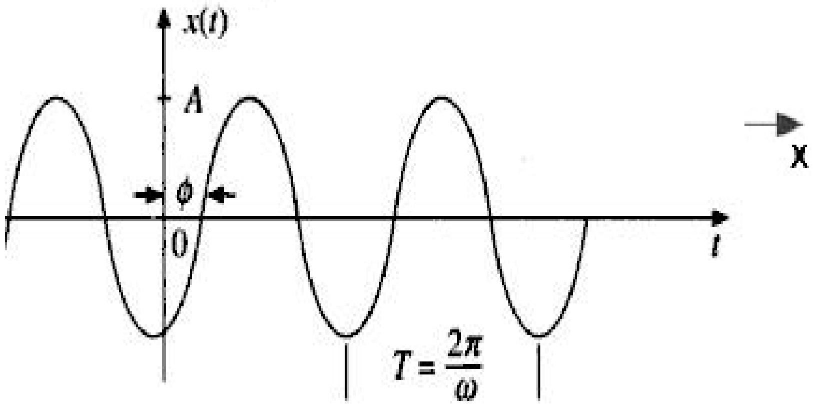
Sonogram

Introduction

- Moving picture as an example of a continuous three dimensional signal



- **Signal modeling :** The representation of a signal by mathematical expression is known as signal modeling.



$$x(t) = A\sin(\omega t + \phi)$$

Continuous-Time and Discrete-Time Signals

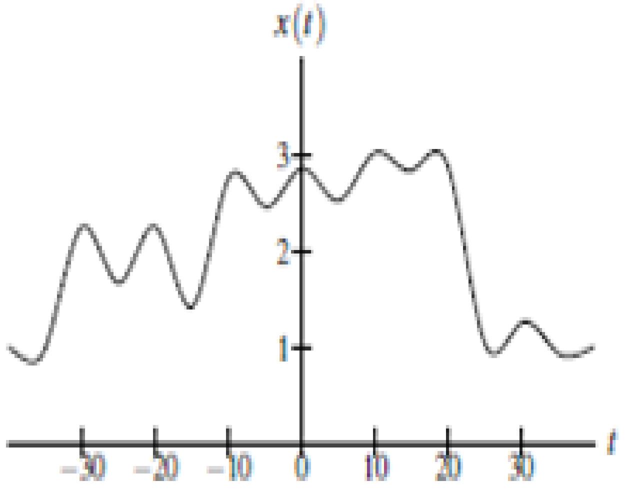
- A signal can also be classified on the basis of whether it is a function of continuous or discrete variables.
- **Continuous-Time Signal**

A signal that is a function of continuous variables (e.g., a real variable) is said to be continuous time.

(or)

Continuous time signals are defined for every value of time t and is represented by x(t). A continuous time signal is also called an analog signal. Most of the signals encountered in practice are continuous – time signals.

-



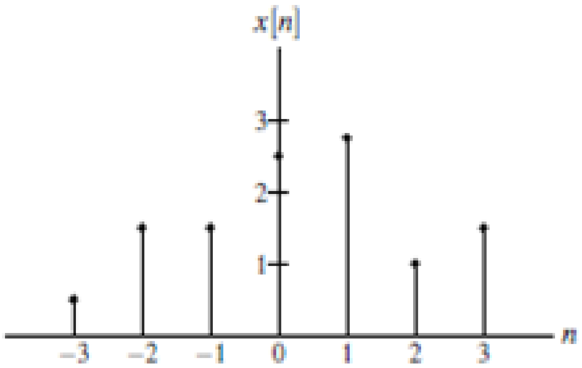
Continuous-Time and Discrete-Time Signals

Discrete-Time Signals

- Similarly, a signal that is a function of discrete variables (e.g., an integer variable) is said to be discrete time.

(or)

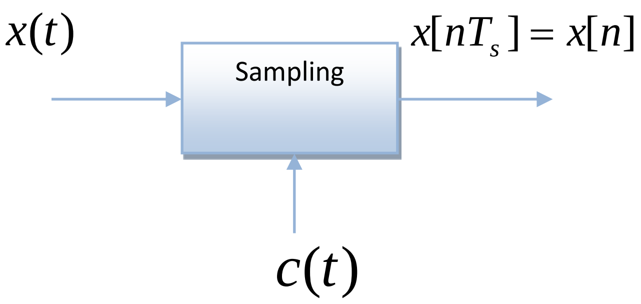
- The discrete time signals are defined at discrete instant of time and is represented by x[n].



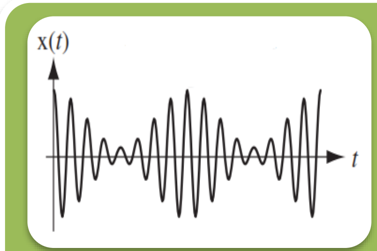
The discrete time signal can be denoted by

$$x[nT_s] = x(t)\big|_{t=nT_s}$$

- where Ts is called sampling period and n is an integer ranging from -∞ to ∞
- x[n] is sampled version of continuous time signal x(t)



Differences between CT,DT and Digital Signals

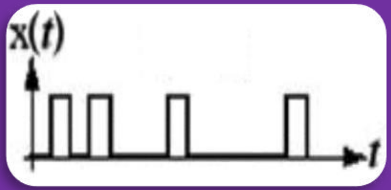


Continuous Time Signal(Analog)

Amplitude is Continuous & Time is also Continuous

Discrete Time Signal

Amplitude is Continuous & Time is Discrete

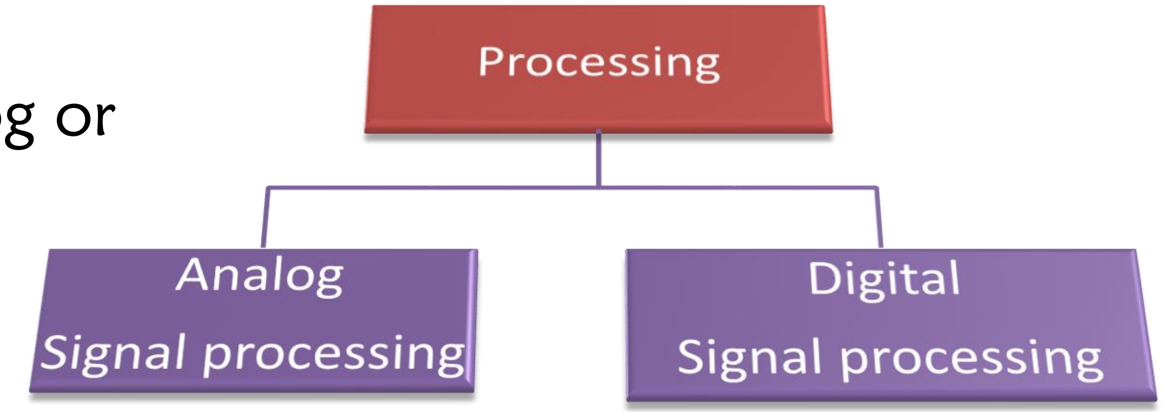


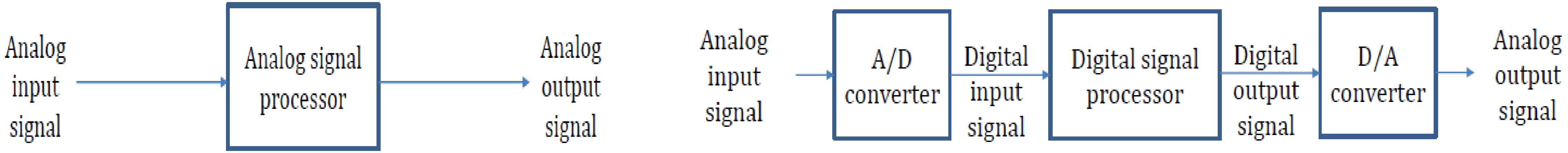
Digital Signal

Amplitude is Discrete & Time is also Discrete

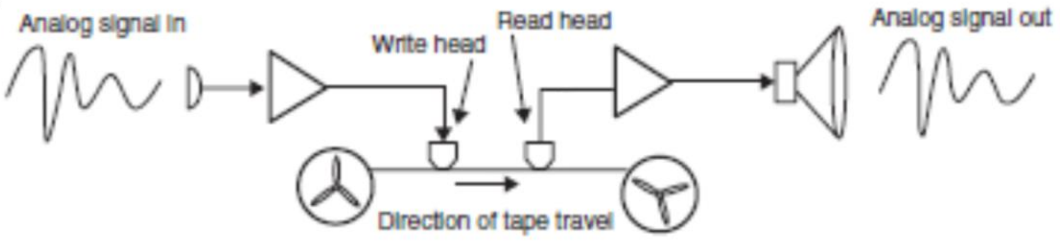
Processing

Processing: To perform operations on data (analog or digital) according to the programmed instructions.

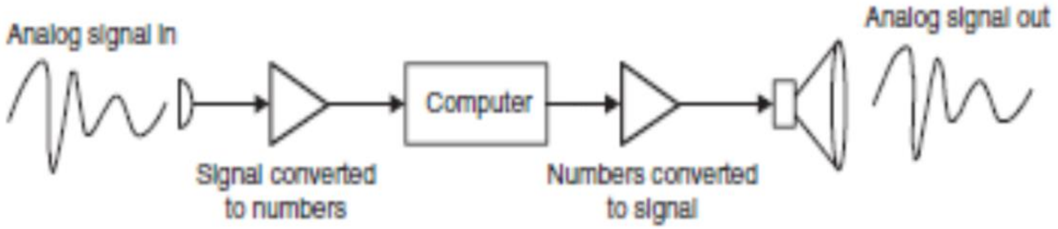




Examples



(a) Analog signal recording.



(b) Digital signal recording.

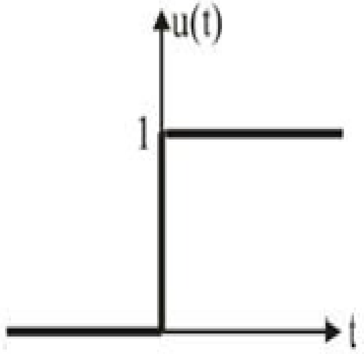
Singularity functions

The Step, Ramp and Impulse functions play a very important role in signal representations. They serve as a basis for representing other signals. These functions are known as singularity functions.

Singularity is a point at which a function does not possess a derivative. Each singularity function has a singular point at the origin and is zero elsewhere.

Unit Step signal:

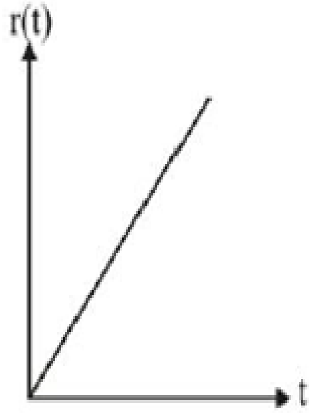
$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



Unit Ramp signal:

$r(t) = t\,u(t)$

$$r(t) = \begin{cases} t\ , & t \geq 0 \\ 0\ , & t < 0 \end{cases}$$



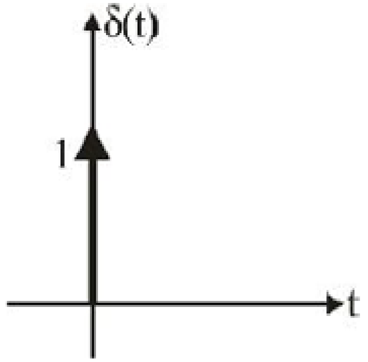
Singularity functions

Singularity functions are also known as Generalized functions.

A generalized function is defined by its effect on other functions instead of by its value at every instant of time.

Impulse signal (Direct Delta Function)

$$\delta(t) = \begin{cases} \infty \;, & t = 0 \\ 0 \;, & t \neq 0 \end{cases} \quad \& \quad \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$



Properties of Impulse Signal

(i) $x(t)\,\delta(t) = x(0)\,\delta(t)$

(ii) $x(t)\,\delta(t - t_o) = x(t_o)\,\delta(t - t_o)$

(iii) $\delta[\alpha(t - \beta)] = \dfrac{1}{|\alpha|}\delta(t - \beta)$

(iv) $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

(v) $\int_{-\infty}^{\infty} x(t)\,\delta(t - t_o) = x(t_o)$

(vi) $x(t) * \delta(t - t_o) = x(t - t_o)$

Fourier Series

The representation of a f(t) over an interval (0, T) by a linear combination of set of infinite number of mutually orthogonal functions is called Fourier Series representation.

Fourier: Periodic signals could be represented by a sum of sinusoids.

If Dirchlet's conditions are satisfied, Fourier series exists

Fourier series is mainly divided into two types:

1. Trigonometric Fourier series Representation
2. Exponential Fourier series Representation

Jean Joseph Baptiste Fourier

Dirichlet's conditions

1. f(t) must be absolutely integrable over the interval T seconds

$$\int_{-T/2}^{T/2} |f(t)| dt < \infty$$

2. The function f(t) must have finite number of discontinuities
3. The function f(t) must remain finite and must have only finite number of maxima and minima.

Trigonometric Fourier Series Representation:

Trigonometric Fourier series Representation means the representation of a f(t) over an interval (0, T)by a linear combination of infinite number of Trigonometric functions.

$$f(t) \approx \cos nw_0 t \,\&\, \sin nw_0 t$$

$$f(t) = a_0 \cos(0)w_0 t + a_1 \cos(1)w_0 t + a_2 \cos(2)w_0 t + \ldots\ldots\ldots + a_n \cos(n)w_0 t + \ldots\ldots\ldots$$

$$+ b_0 \sin(0)w_0 t + b_1 \sin(1)w_0 t + b_2 \sin(2)w_0 t + \ldots\ldots\ldots + b_n \sin(n)w_0 t + \ldots\ldots\ldots$$

$$f(t) = a_0 + a_1 \cos(1)w_0 t + a_2 \cos(2)w_0 t + \ldots\ldots\ldots + a_n \cos(n)w_0 t + \ldots\ldots\ldots$$

$$+ b_1 \sin(1)w_0 t + b_2 \sin(2)w_0 t + \ldots\ldots\ldots + b_n \sin(n)w_0 t + \ldots\ldots\ldots$$

$$\boxed{f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nw_0 t + b_n \sin nw_0 t)}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nw_0 t + b_n \sin nw_0 t)$$

$$a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) dt$$

Component of f(t) at the origin(Average value or DC Value)

$$a_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cos nw_0 t dt$$

Component of f(t)along Cosnwot(Even Component)

$$b_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \sin nw_0 t dt$$

Component of f(t)along Sinnwot(Odd Component)

Exponential Fourier Series Representation:

Exponential Fourier series Representation means the representation of a f(t) over an interval (0,T)by a linear combination of infinite number of exponential functions.

$$f(t) \approx e^{jnw_0t} \qquad (n = 0, \pm1, \pm2, \ldots..)$$

$$f(t) = c_0 e^{j(0)w_0t} + c_1 e^{jw_0t} + c_2 e^{j2w_0t} + \ldots\ldots\ldots\ldots + c_n e^{jnw_0t} + \ldots\ldots\ldots$$

$$c_{-1} e^{-jw_0t} + c_{-2} e^{-j2w_0t} + c_{-3} e^{-j3w_0t} + \ldots. + c_{-n} e^{-jnw_0t} + \ldots\ldots\ldots$$

$$\boxed{f(t) = c_0 + \sum_{n=1}^{\infty} c_n e^{jnw_0t} + \sum_{n=1}^{\infty} c_{-n} e^{-jnw_0t}}$$

$$\boxed{f(t) = c_0 + \sum_{n=1}^{\infty} c_n e^{jnw_0t} + \sum_{n=-1}^{-\infty} c_n e^{jnw_0t}}$$

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jnw_0t}}$$

$$\boxed{f(t) = c_0 + \sum_{n=1}^{\infty} c_n e^{jnw_0 t} + \sum_{n=1}^{\infty} c_{-n} e^{-jnw_0 t}}$$

$$c_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)dt$$

$$c_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)e^{-jnw_0 t}\, dt$$

$$c_{-n} = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)e^{jnw_0 t}\, dt$$

Component of f(t) at the origin(Average value or DC Value)

Component of f(t)along $e^{jnw_0 t}$

Component of f(t)along $e^{-jnw_0 t}$

Relation between Trigonometric Fourier series and Exponential Fourier series

Exponential Fourier Series coefficients in terms of Trigonometric Fourier Series coefficients

$$c_0 = a_0$$

$$c_n = \frac{a_n - jb_n}{2}$$

$$c_{-n} = \frac{a_n + jb_n}{2}$$

Trigonometric Fourier series coefficients in terms of Exponential Fourier Series coefficients

$$a_0 = c_0$$

$$a_n = c_n + c_{-n}$$

$$b_n = j\left[c_n - c_{-n}\right]$$

Convergence of Fourier series

1. f(t) must be absolutely integrable over the interval T seconds

   $$\int_{-T/2}^{T/2} |f(t)|dt < \infty$$

   This is known as weak Dirichlet's condition.

2. The function f(t) must have finite number of discontinuities

3. The function f(t) must remain finite and must have only finite number of maxima and minima.

   These are known as strong Dirichlet's conditions.

Important Points on Fourier Series

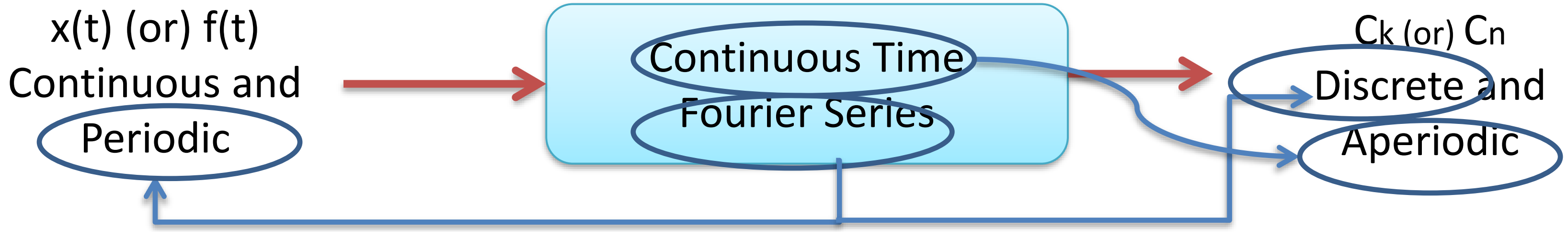
1. If the function is an **even function** then the corresponding FS contains only **Even components** and Odd components will be zero.

2. If the function is an **odd function** then the corresponding FS contains only **Odd components** and Even components will be zero.

3. If the function exhibits **Hidden symmetry** then the corresponding FS contains **either Even** components **or Odd** components.

4. If the function exhibits **Rotational (Half wave) symmetry** then the corresponding FS contains only. **Odd harmonics(both even and odd)**

Spectrum of Continuous Time Fourier Series(CTFS)



Key Points:

1.Fourier Series is applicable only for **Periodic Signals**

2.Fourier Series always generates **Discrete spectrum.**

3.If the input signal is in **Continuous Time** ,then the spectrum always **Aperiodic.**

Problems on Fourier Series

1.The trigonometric Fourier series of an even function of time does not have ……….
[GATE 2011: 1 Mark]

(a) the dc term
(b) cosine terms
(c) sine terms
(d) odd harmonic terms

Solution:

For periodic even function, the trigonometric Fourier series does not contain the sine terms (odd functions) It has dc term and cosine terms of all harmonics.

Option(c)

Problems on Fourier Series

2.The trigonometric Fourier series of a periodic time function can have only........
[GATE 1998: 1 Mark]

(a) cosine terms
(b) sine terms
(c) cosine and sine terms
(d) dc and cosine terms

Solution:

$$x(t) = \sum_{n=0}^{\infty} a_n \cos n\,\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\,\omega_0\, t$$

Thus the series has cosine terms of all harmonics: $n\omega_0$, n=0,1,2−−−
Where 0th harmonic = dc term (average or mean) = a_0 and sine terms of all harmonics: $n\omega_0$, n=1,2,−−−−.
Option : (c)

Problems on Fourier Series

3.The Fourier series of an odd periodic function, contains only

(a) odd harmonics
(b) even harmonics
(c) cosine terms
(d) sine terms

[GATE 1994: 1 Mark]

Solution:

If periodic function is odd the dc term a_0 =0 and also cosine terms (even symmetry)

It contains only sine terms

Option (d)

Problems on Fourier Series

4. The Fourier series of a real periodic function has only

P. Cosine terms if it is even

Q. Sine terms if it is even

R. Cosine terms if it is odd

S. Sine terms if it is odd

Which of the above statements are correct? **[GATE 2009: 1 Mark]**

(a) P and S
(b) P and R
(c) Q and S
(d) Q and R

Solution:

The Fourier series for a real periodic function has only cosine terms if it is even and sine terms if it is odd

Option (a)

Problems on Fourier Series

5.Which of the following cannot be the Fourier series expansion of a periodic signals?

(a) $x(t)=2\cos t+3\cos 3t$

(b) $x(t)=2\cos\pi t+7\cos t$

(c) $x(t)=\cos t+0.5$

(d) $x(t)2\cos 1.5\pi t+\sin 3.5\pi t$

[GATE 2002: 1 Mark]

Solution:

(a) $x(t)=2\cos t+3\cos t$ is periodic signal with fundamental frequency $\omega 0=1$

(b) $x(t)=2\cos\pi t+7\cos t$ The frequency of first term $\omega 1=\pi$ frequency of 2nd term is $\omega 2=1$

$\omega 1/\omega 2=\pi/1$ *is not the rational number*

So $x(t)$ is aperiodic or not periodic

(c) $x(t)=\cos t+0.5$ is a periodic function with $\omega 0=1$

(d) $x(t)=2\cos(1.5\pi)t+\sin(3.5\pi)t$ first term has frequency $\omega 1=1.5\pi$ 2nd term has frequency $\omega 2=3.5\pi$

$\omega 1/\omega 2=1.5\pi/3.5\pi=1.5/3.5=(3\times0.5)/(7\times0.5)=3/7$

So about ratio is rational number $x(t)$ is a periodic signal, with fundamental frequency $\omega 0=0.5\pi$

Since function in (b) is non periodic. So does not satisfy Dirictilet condition and cannot be expanded in Fourier series

Problems on Fourier Series

6. Choose the function $f(t)$ $,-\infty<t<\infty$, for which a Fourier series cannot be defined.

(a) $3\sin(25t)$

(b) $4\cos(20t+3)+2\sin(710t)$

(c) $\exp(-|t|)\sin(25t)$

(d) 1

[GATE 2005: 1 Mark]

Solution:

Fourier series is defined for periodic function and constant

(a) $3\sin(25\ t)$ *is periodic* $\omega=25$

(b) $4\cos(20\ t+3)+2\sin(710\ t)$ sum of two periodic function is also periodic function

(c) $e-|t|\sin 25\ t$ Due to decaying exponential decaying function it is not periodic. So Fourier series cannot be defined for it.

(d) Constant, Fourier series exists.

Fourier series can’t be defined for option (c)

Problems on Fourier Series

7.The Fourier series expansion of a real periodic signal with fundamental frequency f_0 is given by

$$g_p(t) = \sum_{n=-\infty}^{\infty} C_n \, e^{j\,2\pi n\, f_0\, t}$$

It is given that $C_3 = 3 + j5$ *then* C_{-3} *is*

(a) $5+3j$

(b) $-3-j5$

(c) $-5+3j$

(d) $3-j5$ **[GATE 2003: 1 Mark]**

Solution:

Given $C_3 = 3 + j5$

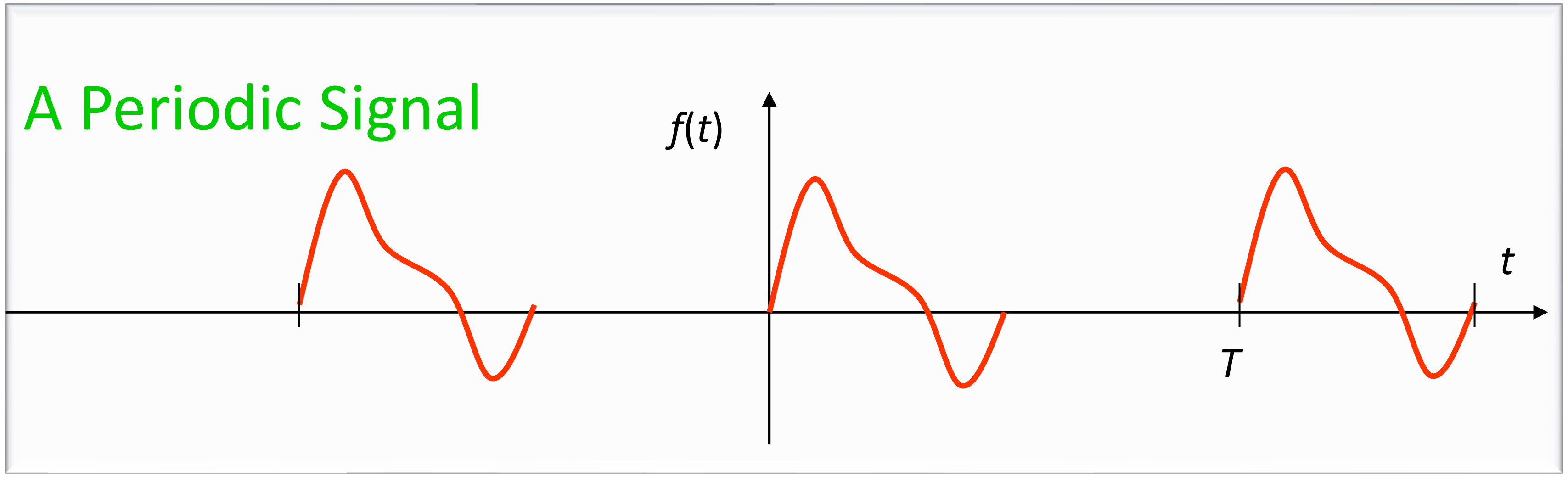
We know that for real periodic signal

$C_{-k} = C_k{}^*$

$So,\ C_{-3} = C_3{}^* = (3 - j5)$

Option (d)

How to Deal with Aperiodic Signal?



If $T \to \infty$, what happens?

Fourier Transforms

The main drawback of Fourier series is, it is only applicable to periodic signals. There are some naturally produced signals such as nonperiodic or aperiodic, which we cannot represent using Fourier series.

The magnitude spectrum of an aperiodic signal is not a line spectrum (as with a periodic signal), but instead occupies a continuum of frequencies.

To overcome this shortcoming, Fourier developed a mathematical model to transform signals between time (or spatial) domain to frequency domain & vice versa, which is called 'Fourier transform'.

Fourier transform has many applications in physics and engineering such as analysis of LTI systems, RADAR, astronomy, signal processing etc.

Fourier Transform Pair

Fourier Transform:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$$

Analysis Equation

Inverse Fourier Transform:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega$$

Synthesis Equation

Note: Remember $\omega = 2\pi f$

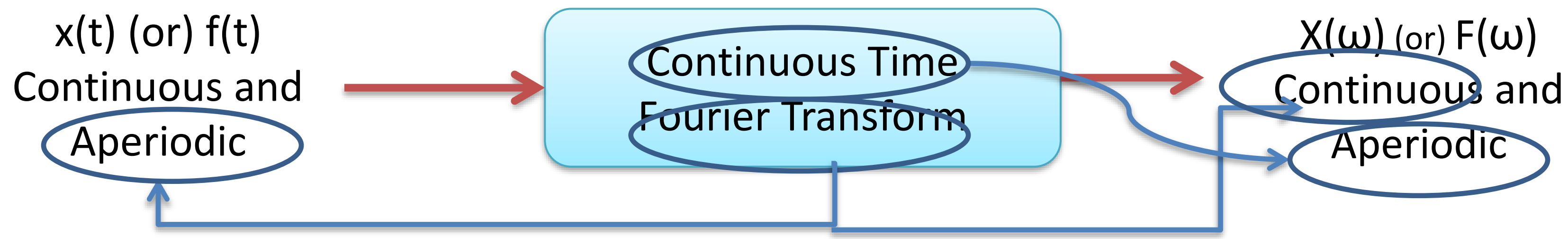
Existence Fourier Transform

1. f(t) should be absolutely integrable over the interval -∞ to ∞

$$\int_{-\infty}^{\infty} |f(t)| dt < \infty$$

2. f(t) has a finite number of discontinuities in every finite time interval. Further, each of these discontinuities must be finite
3. f(t) has a finite number of maxima and minima in every finite time interval.

Spectrum of Continuous Time Fourier Transform(CTFT)



Key Points:

1.**Fourier Transform** is applicable only for Ap**eriodic Signals**

2.**Fourier Transform** always generates **Continuous spectrum.**

3.If the input signal is in **Continuous Time** ,then the spectrum always **Aperiodic.**

Properties of Fourier Transforms

1.Linearity

$$a_1 f_1(t) + a_2 f_2(t) \xleftrightarrow{\mathcal{F}} a_1 F_1(\omega) + a_2 F_2(\omega)$$

2.Time Reversal

$$f(-t) \xleftrightarrow{\mathcal{F}} F\left(-\omega\right)$$

Properties of Fourier Transforms

3.Time Scaling

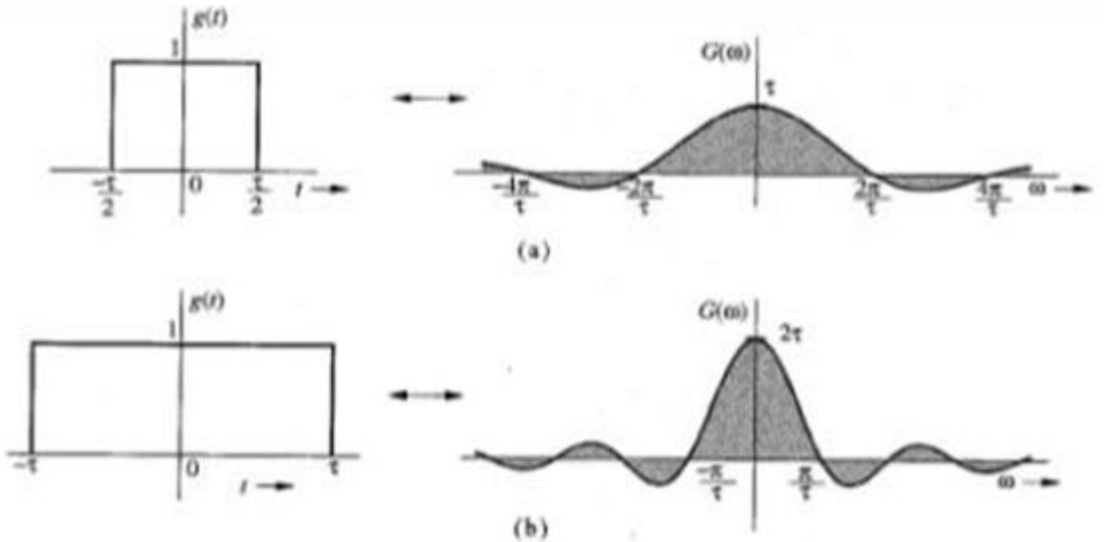
$$f(at) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$$

Significance of Scaling property

Time compression of a signal results in **spectral expansion**

and

time expansion of a signal results in **spectral compression.**



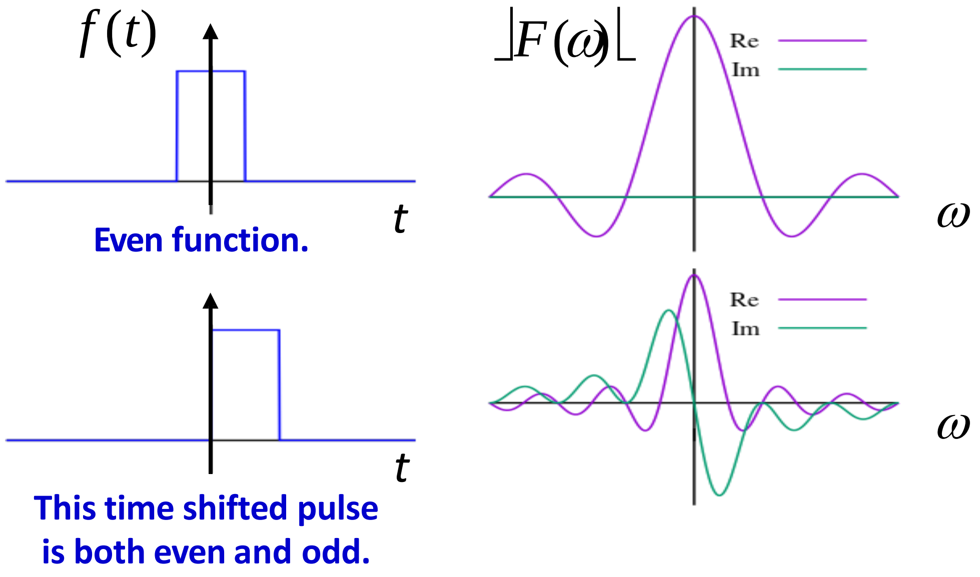
Properties of Fourier Transforms

4.Time Shifting

$$f(t-t_0) \xleftrightarrow{\mathcal{F}} F(\omega)e^{-j\omega t_0}$$

Significance of Scaling property

Delaying a signal by t_0 seconds does not change its amplitude spectrum, but the phase spectrum is changed by $-2\pi f t_0$.
Note that the phase spectrum shift changes linearly with frequency f.



Properties of Fourier Transforms

5.Frequency Shifting Property

(Modulation or Frequency Translation)

$$f(t)e^{j\omega_0} \xleftrightarrow{\mathcal{F}} F[(\omega-\omega_0)]$$

$$f(t)e^{-j\omega_0} \xleftrightarrow{\mathcal{F}} F[(\omega+\omega_0)]$$

Frequency Shifting Property is Very Useful in Communications:

Modulation

Special Application :

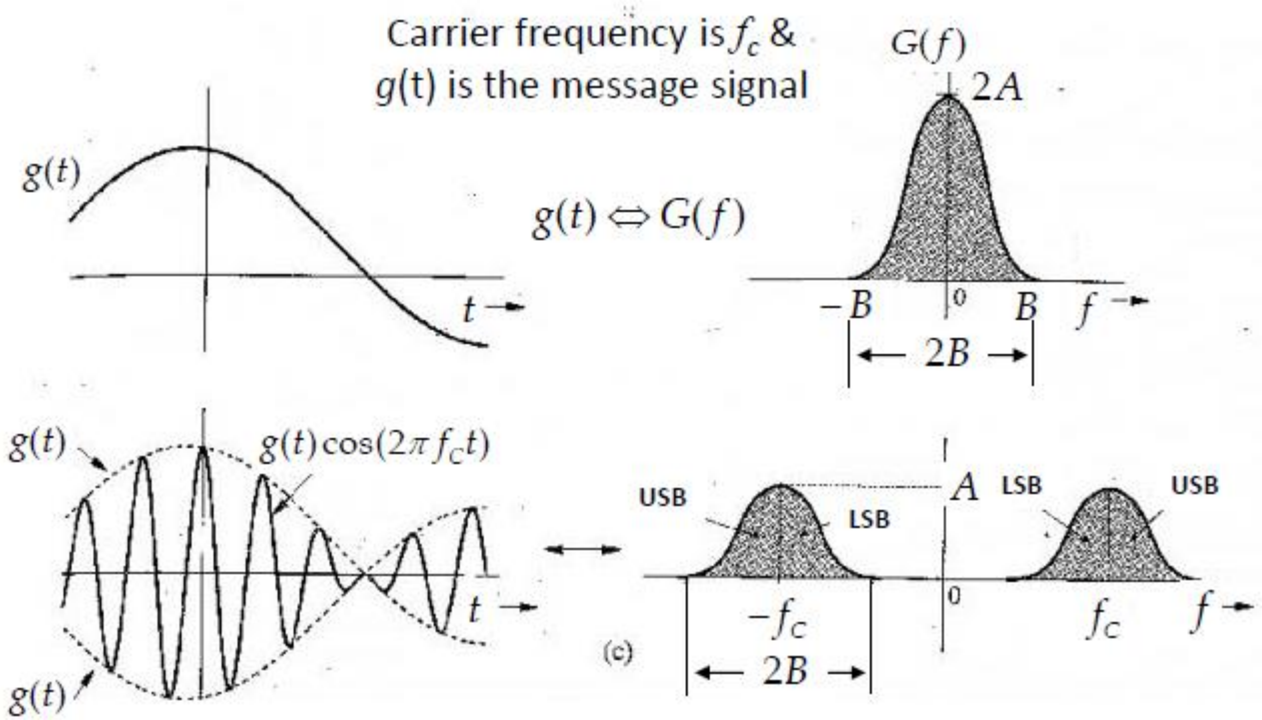
$$\cos(\omega_o t) = \frac{1}{2}(e^{+j\omega_0 t} + e^{-j\omega_0 t});$$

$$f(t)\cos(\omega_o t) \xleftrightarrow{\mathcal{F}} \frac{1}{2}(F[(\omega+\omega_0)] + F[(\omega-\omega_0)])$$

Properties of Fourier Transforms

Significance of Frequency Shifting property

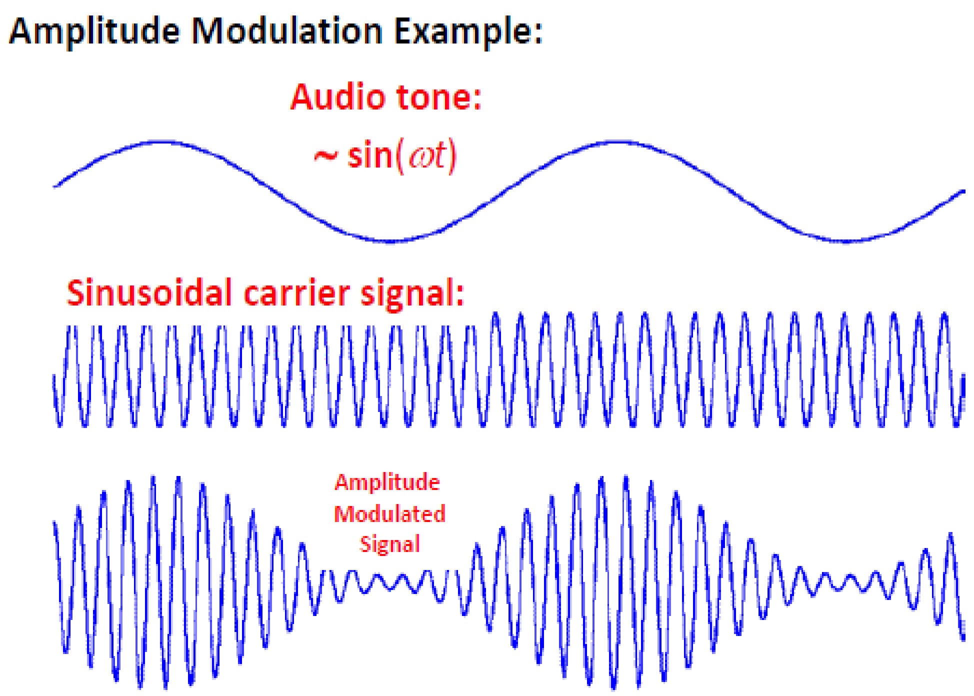
Multiplication of a signal *g(t)* by the factor [cos($2\pi f_C t$)] places *G(f)* centered at $f = \pm f_C$.



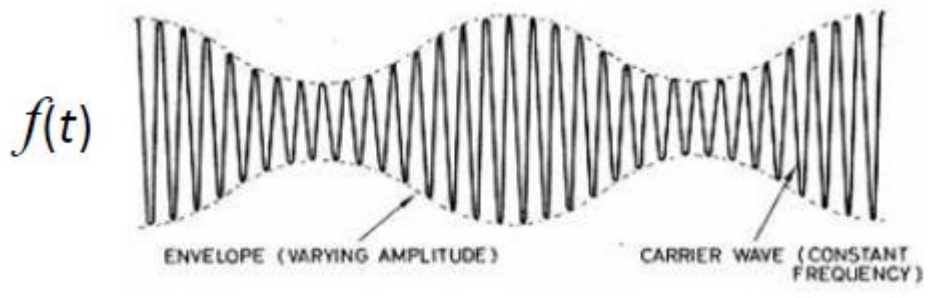
Courtesy: B. P Lathi & Z. Ding, 4th ed., Chapter 4, Section 4.2, Figure 4.1 (p. 181)

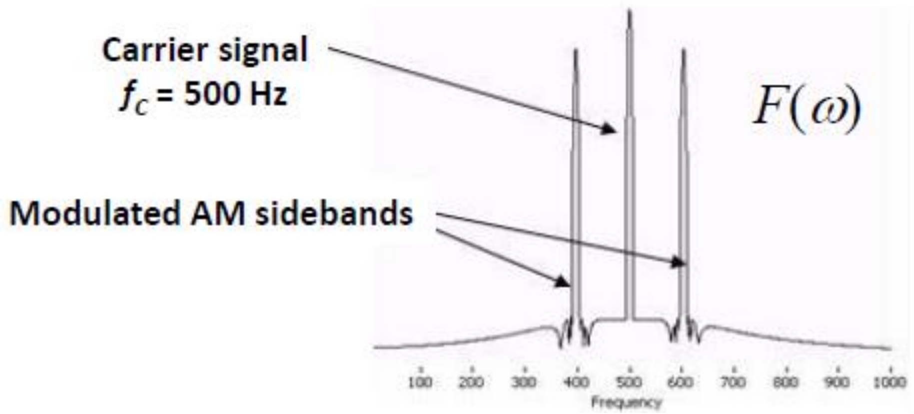
Properties of Fourier Transforms

Modulation Comes from Frequency Shifting property



FT of AM tone Modulated Signal





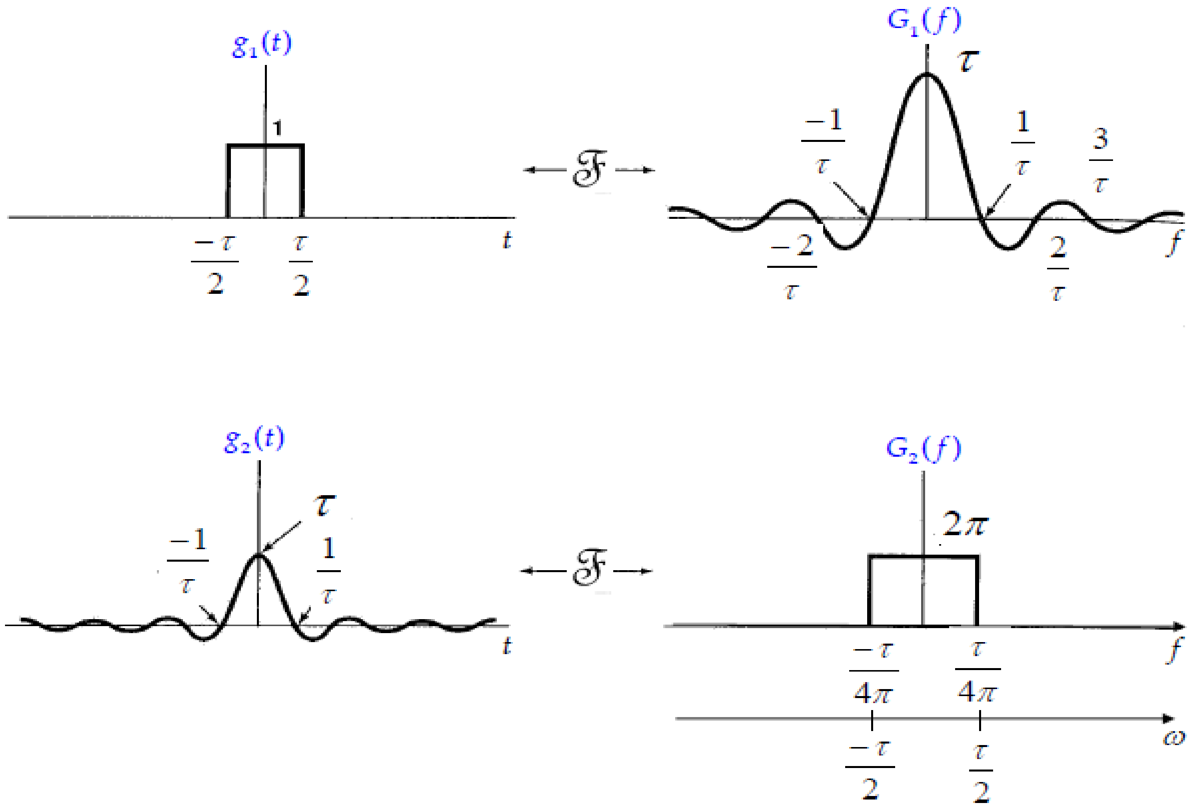
Courtesy :http://www.ni.com/tutorial/5421/en/

Properties of Fourier Transforms

6.Symmetry(Duality)Property

$$\mathcal{F}[F(t)] = 2\pi f(-\omega)$$

Significance



Properties of Fourier Transforms

7.Time Differentiation Property:

$$\frac{df(t)}{dt} \xleftrightarrow{\mathcal{F}} j\omega F(\omega)$$

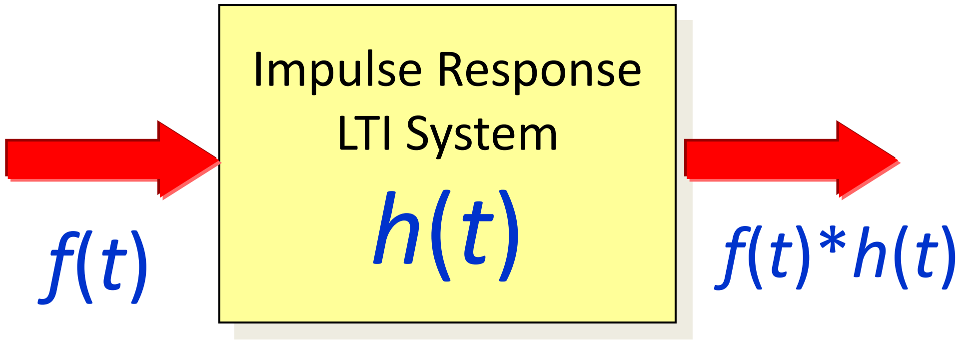
$$f^{(n)}(t) \xleftrightarrow{\mathcal{F}} (j\omega)^n F(\omega)$$

8. Frequency Differentiation Property

$$\mathcal{F}[-jtf(t)] \xleftrightarrow{\mathcal{F}} \frac{dF(\omega)}{d\omega}$$

Basics on Convolution

Unit Impulse Response



Convolution Definition

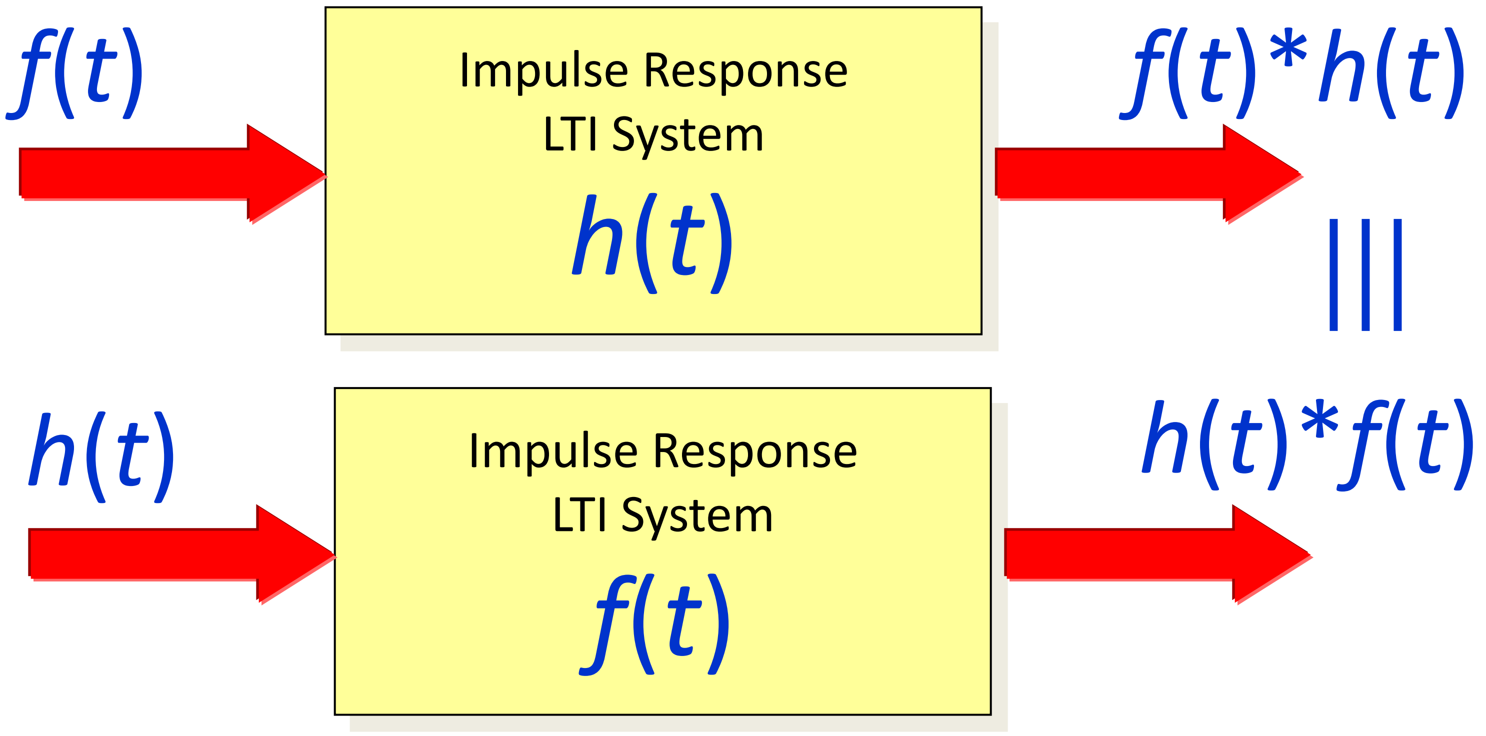
The convolution of two functions f1(t) and f2(t) is defined as:

$$f(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau) d\tau$$

$$= f_1(t) * f_2(t)$$

Properties of Convolution

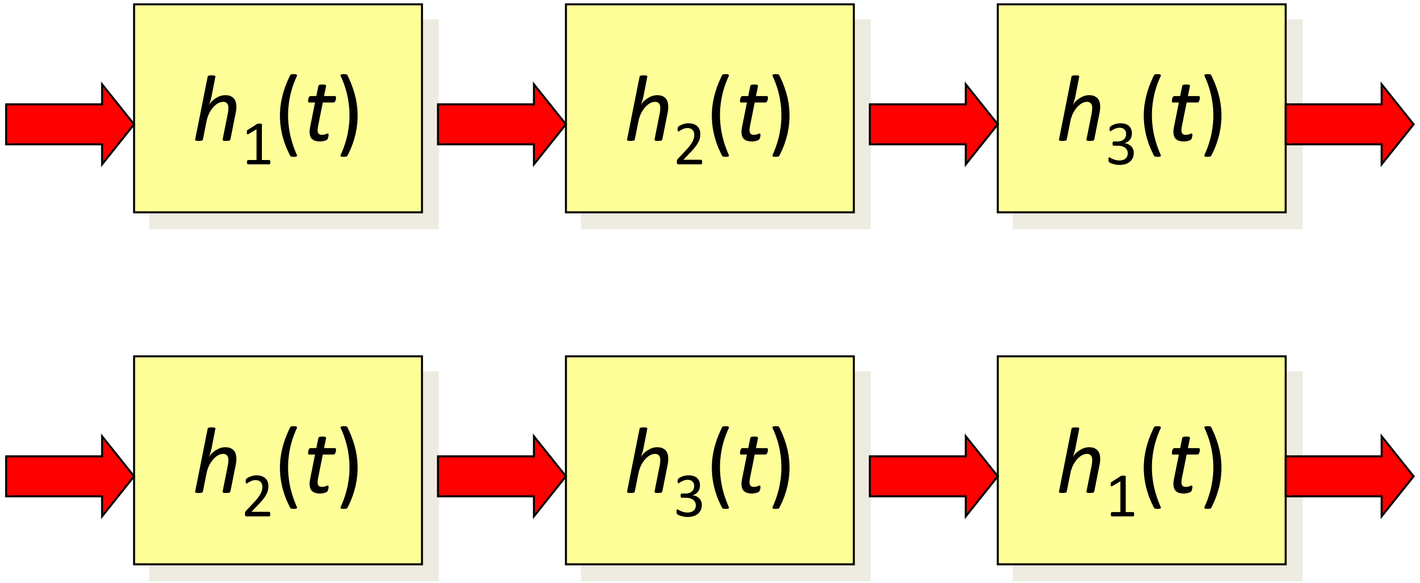
$$1.\ f_1(t) * f_2(t) = f_2(t) * f_1(t)$$



Properties of Convolution

$$2.[f_1(t) * f_2(t)] * f_3(t) = f_1(t) * [f_2(t) * f_3(t)]$$

The following two systems are identical



Properties of Convolution

$$\boxed{4.\ f(t) * \delta(t) = f(t)}$$

$$f(t) \longrightarrow \boxed{\delta(t)} \longrightarrow f(t)$$

$$f(t) * \delta(t) = \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} f(t-\tau)\delta(\tau)d\tau$$

$$= f(t)$$

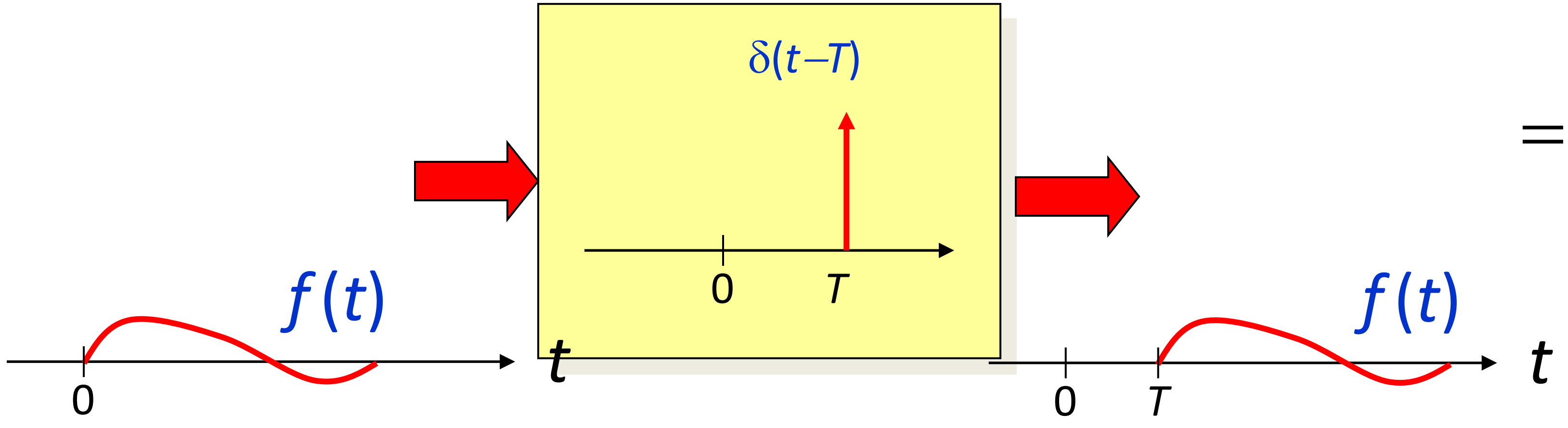
Properties of Convolution

$$5.\ f(t) * \delta(t-T) = f(t-T)$$

$$f(t) * \delta(t-T) = \int_{-\infty}^{\infty} f(\tau)\delta(t-T-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} f(t-T-\tau)\delta(\tau)d\tau$$

$$= f(t-T)$$



Properties of Fourier Transforms

9.Convolution Property: (a) Time Convolution Property:

$$\boxed{f_1(t) * f_2(t) \xleftrightarrow{\mathcal{F}} F_1(\omega)F_2(\omega)}$$

$f(t)$ → Impulse Response LTI System $h(t)$ → $f(t)*h(t)$

$F(\omega)$ → Impulse Response LTI System $H(\omega)$ → $F(\omega)H(\omega)$

Convolution in Time domain leads to multiplication in Frequency domain

Properties of Fourier Transforms

9.Convolution Property: (b) Frequency Convolution Property:

$$\boxed{f_1(t)\, f_2(t) \xleftrightarrow{\mathcal{F}} \frac{1}{2\pi} F_1(j\omega) * F_2(j\omega)}$$

Convolution in Frequency domain leads to multiplication in Time domain

Properties of Convolution

10.Parseval's (Energy)Theorem:

$$\int_{-\infty}^{\infty} |\, f(t)\,|^2 dt \xleftrightarrow{\mathcal{F}} \frac{1}{2\pi}\int_{-\infty}^{\infty} |\, F(\omega)\,|^2 d\omega$$

Parseval's theorem that the signal energies of an energy signal and its FT are equal

Fourier Transforms of Some useful functions

Function	Time Domain	Frequency domain
1.Unit Impulse	$\delta(t) = \begin{cases} 1 \text{ for } t = 0 \\ 0 \text{ for } t \neq 0 \end{cases}$	1
2. Single Sided Real exponential function	$e^{-at}u(t)$	$\dfrac{1}{a + j\omega}$
3. Double sided real exponential function	$\mathbf{e^{-a\|t\|}}$	$\dfrac{2a}{a^2 + \omega^2}$
4.Complex Exponential Function	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
5. Constant Amplitude	1	$2\pi\delta(\omega)$
6. Signum function	$\mathrm{sgn}(t) = \begin{cases} 1 & \text{for } t > 0 \\ -1 & \text{for } t < 0 \end{cases}$	$\dfrac{2}{j\omega}$

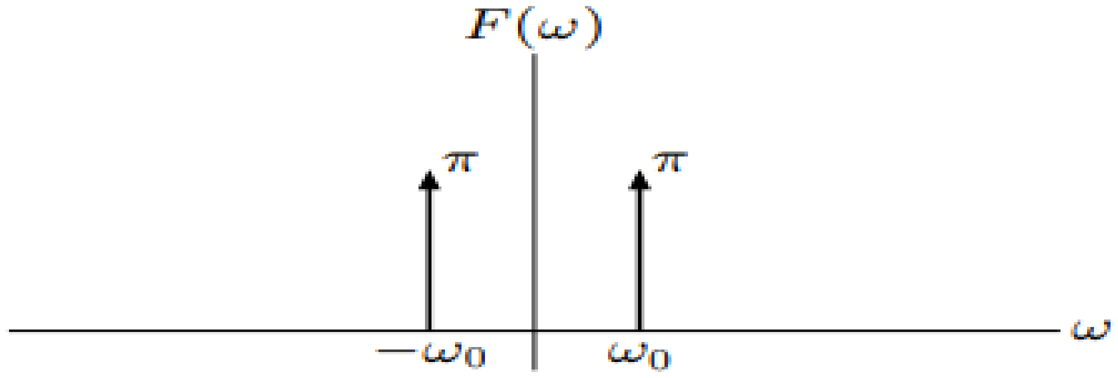
Fourier Transforms of Some useful functions

Function	Time Domain	Frequency domain
7. Unit step function	$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$	$\pi\delta(\omega) + \frac{1}{j\omega}$
8.Rectangular pulse (Gate pulse)	$\mathrm{rect}\left(\frac{t}{\tau}\right) = \Pi\left(\frac{t}{\tau}\right) = \begin{cases} 1 & \text{for } \|t\| < \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$	$\tau \, sinc \, \omega(\tau/2)$
9.Triangular Pulse	$\Delta\left(\frac{t}{\tau}\right) = \begin{cases} 1 - \frac{2\|t\|}{\tau} & \text{for } \|t\| < \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$	$\frac{\tau}{2} sinc^2\left(\frac{\omega\tau}{4}\right)$
10. Cosine wave	$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_o) + \delta(\omega + \omega_o)]$
11. Sine wave	$\sin\omega_0 t$	$-j\pi[\delta(\omega - \omega_o) - \delta(\omega + \omega_o)]$

Problems on Fourier Transforms

sinusoidal signals: Fourier transform of $f(t) = \cos\omega_0 t$

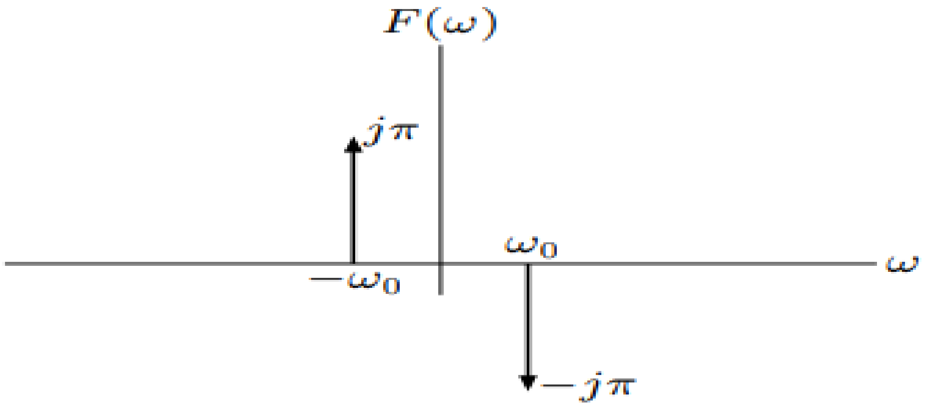
$$\begin{aligned} F(\omega) &= \frac{1}{2}\int_{-\infty}^{\infty} \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) e^{-j\omega t}\, dt \\ &= \frac{1}{2}\int_{-\infty}^{\infty} e^{-j(\omega-\omega_0)t} dt + \frac{1}{2}\int_{-\infty}^{\infty} e^{-j(\omega+\omega_0)t} dt \\ &= \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0) \end{aligned}$$



Problems on Fourier Transforms

Fourier transform of $f(t) = \sin \omega_0 t$

$$
\begin{aligned}
F(\omega) &= \frac{1}{2j}\int_{-\infty}^{\infty} \left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right) e^{-j\omega t}\, dt \\
&= \frac{1}{2j}\int_{-\infty}^{\infty} e^{-j(\omega-\omega_0)t} dt + -\frac{1}{2j}\int_{-\infty}^{\infty} e^{-j(\omega_0+\omega)t} dt \\
&= -j\pi\delta(\omega-\omega_0) + j\pi\delta(\omega+\omega_0)
\end{aligned}
$$



Problems on Fourier Transforms

1.The Fourier transform of a real valued time signal has
(a) odd symetry
(b) even symmetry
(c) conjugate symmetry
(d) no symmetry
[GATE 1996: 1 Mark]

Solution:

If $f(t)$ is a real function, and $F(j\omega) = F_R(j\omega) + jF_I(j\omega)$

$$F(-j\omega) = F^*(j\omega)$$

$F_R(j\omega)$ is even, and $F_I(j\omega)$ is odd.

$$\underbrace{F_R(-j\omega) = F_R(j\omega)} \qquad \underbrace{F_I(-j\omega) = -F_I(j\omega)}$$

For real valued time signal, Fourier Transform has conjugate symmetry

Option (c)

Problems on Fourier Transforms

2. A signal x(t) has a Fourier transform X(ω). If x(t) is a real and odd function of t, then X(ω) is

(a) a real and even function of ω

(b) an imaginary and odd function of ω

(c) an imaginary and even function of ω

(d) a real and odd function of ω

[GATE 1999: 1 Mark]

Solution:

If $f(t)$ is real and even then $F(\omega)$ is real

$$\text{Even} \rightarrow f(t) = f(-t)$$

$$F(\omega) = F(-\omega)$$

$$\text{Real} \rightarrow f(-\omega) = f^*(\omega)$$

$$\text{Or } F(\omega) = F^*(\omega)$$

If $f(t)$ is real and odd

$F(\omega)$ is pure imaginary

$$\text{odd} \rightarrow f(t) = -f(-t)$$

$$F(\omega) = -F(-\omega)$$

Option (b)

Problems on Fourier Transforms

3.If $G(f)$ represents the Fourier Transform of a signal $g(t)$which is real and odd symmetric in time, then

(a) G(f) is complex

(b) G(f) is imaginary

(c) G(f) is real

(d) G(f) is real

[GATE 1992: 2 Marks]

Solution:

$$g(t) \rightarrow G(f)$$

Note, If $g(t)$ is real and even,

$G(f)$ is also real and even

But if $g(t)$ is real and odd

$G(f)$ is imaginary and odd

Option (b)

Problems on Fourier Transforms

4.The amplitude spectrum of a Gaussian pulse is

(a) uniform

(b) a sine function

(c) Gaussian

(d) An impulse function

[GATE 1998: 1 Mark]

Solution:

Gaussian pulse is defined by

$$f(t) = e^{-\pi t^2}$$

Fourier Transform of this pulse can be evaluated

$$\mathcal{F}[e^{-\pi t^2}] = \int_{-\infty}^{\infty} e^{-\pi t^2} \; e^{-j\omega t} \, dt$$

After evaluation of integral

$$\mathcal{F}[e^{-\pi t^2}] = e^{-\pi f^2}$$

Option (C)

Problems on Fourier Transforms

5.The Fourier transform of a voltage signal x(t) is X(f). The unit of $|X(f)|$ is

(a) Volt

(b) Volt – sec

(c) Volt / sec

(d) $Volt^2$

[GATE 1998: 1 Mark]

Solution:

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

Looking at R.H.S expression, then unit of X(f) will be volt – sec

Option (b)

Problems on Fourier Transforms

6.If a signal $f(t)$ has energy E, the energy of the signal $f(2t)$ is equal to

(a) E

(b) E/2

(c) 2E

(d) 4E

[GATE 2001: 1 Mark]

Solution:

Given,

Signal $f(t)$ has energy E.

Find energy of the signal $f(2t)$.

Energy of signal

$$f(t) = \int_{-\infty}^{\infty} f^2(t)dt$$

So, energy of signal $f(2t)$ will be

$$= \int_{-\infty}^{\infty} f^2(2t)\ dt$$

$$= \int_{-\infty}^{\infty} f^2(\tau)\,\frac{d\tau}{2} = \frac{E}{2}$$

Option (b)

Hilbert Transform

- Fourier, Laplace, and z-transforms **change from the time-domain** representation of a signal **to the frequency-domain** representation of the signal
- The resulting two signals are **equivalent representations of the same signal** in terms of time or frequency
- In contrast, The Hilbert transform does **not involve a change of domain**, unlike many other transforms

Strictly speaking, the Hilbert transform is not a transform in this sense

- First, the result of a Hilbert transform is not equivalent to the original signal, rather it is a completely different signal
- Second, the Hilbert transform does not involve a domain change, i.e., the Hilbert transform of a signal x(t) is another signal denoted by in the same domain (i.e.time domain)

- The Hilbert transform of a signal x(t) is a signal $\hat{x}(t)$ whose frequency components lag the frequency components of x(t) by 90°
  - $\hat{x}(t)$ has exactly the same frequency components present in x(t) with the same amplitude–except there is a 90° phase delay
  - The Hilbert transform of

    $x(t) = A\cos(2\pi f_0 t + \theta)$ is $A\cos(2\pi f_0 t + \theta - 90°) = A\sin(2\pi f_0 t + \theta)$
  - The Hilbert transform of

    $x(t) = A\sin(2\pi f_0 t + \theta)$ is $A\sin(2\pi f_0 t + \theta - 90°) = -A\cos(2\pi f_0 t + \theta)$

$e^{j2\pi f0t}$ will become $e^{j2\pi f_0 t-\frac{\pi}{2}} = -je^{j2\pi f_0 t}$

$e^{-j2\pi f0t}$ will become $e^{-j(2\pi f_0 t-\frac{\pi}{2})} = je^{j2\pi f_0 t}$

At positive frequencies, the spectrum of the signal is multiplied by *-j*
At negative frequencies, it is multiplied by *+j*

This is equivalent to saying that the spectrum (Fourier transform) of the signal is multiplied by *-j*sgn(*f*).

Assume that x(t) is real and has no DC component : $X(f)|_{f=0} = 0$, then

$$F\left[\hat{x}(t)\right] = -j\,\mathrm{sgn}(f)X(f)$$

$$F^{-1}\left[-j\,\mathrm{sgn}(f)\right] = \frac{1}{\pi t}$$

$$\hat{x}(t) = \frac{1}{\pi t} * x(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\,d\tau$$

The operation of the Hilbert transform is equivalent to a convolution, i.e., filtering

- Obviously performing the Hilbert transform on a signal is equivalent to a 90° phase shift in all its frequency components
- Therefore, the only change that the Hilbert transform performs on a signal is changing its phase
- The amplitude of the frequency components of the signal do not change by performing the Hilbert-transform
- On the other hand, since performing the Hilbert transform changes cosines into sines, the Hilbert transform $\hat{x}(t)$ of a signal x(t) is orthogonal to x(t)
- Also, since the Hilbert transform introduces a 90° phase shift, carrying it out twice causes a 180° phase shift, which can cause a sign reversal of the original signal

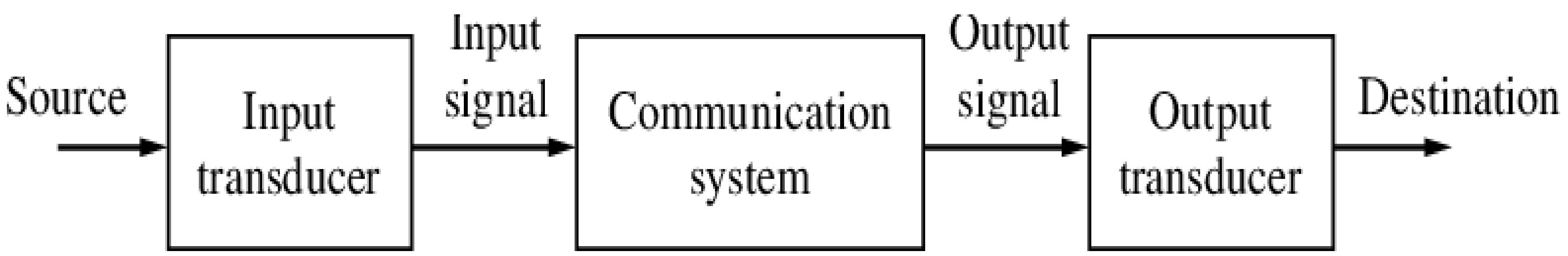
Properties Of Hilbert Transform

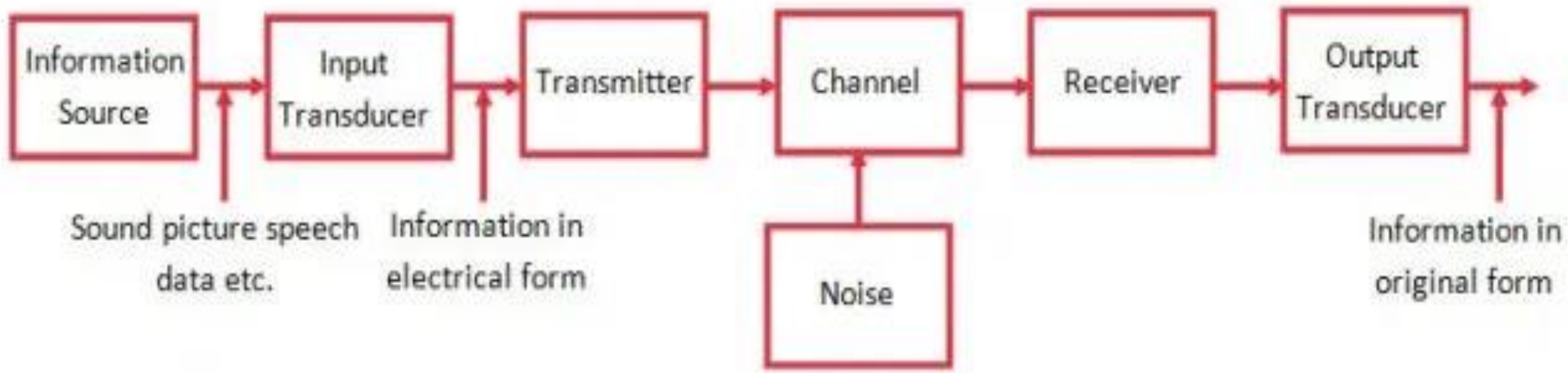
a. If $x(t) = x(-t)$, then $\hat{x}(t) = -\hat{x}(-t)$.
b. If $x(t) = -x(-t)$, then $\hat{x}(t) = \hat{x}(-t)$.
c. If $x(t) = \cos\omega_0 t$, then $\hat{x}(t) = \sin\omega_0 t$.
d. If $x(t) = \sin\omega_0 t$, then $\hat{x}(t) = -\cos\omega_0 t$.
e. $\hat{\hat{x}}(t) = -x(t)$
f. $\int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} \hat{x}^2(t)\,dt$
g. $\int_{-\infty}^{\infty} x(t)\hat{x}(t)\,dt = 0$

A signal x(t) and its Hilbert transform $\hat{x}(t)$ have

1. the same amplitude spectrum
2. the same autocorrelation function
3. x(t) and $\hat{x}(t)$ are orthogonal
4. The Hilbert transform of $\hat{x}(t)$ is -x(t)

Elements of an Electrical Communication system





Need for Modulation

- The purpose of a communication system is to transmit information bearing signals or baseband signals through a communication channel separating the Tx. from the Rx.
- Baseband signal – Used to represent the band of frequencies in the original signal as delivered by the source of information
- Communication channel requires Shift of the range of Baseband frequencies into other frequency ranges suitable for transmission and corresponding shift back to the original frequency range after reception
- Radio system – Operates at 30 KHz and above, whereas as baseband signals usually contains frequencies in AF range
- Hence, some form of frequency – band shifting must be used for satisfactory operation of the system
- A shift of the range of frequencies in a signal is accomplished by ‘Modulation’

Need for Modulation

- Modulation permits the use of Multiplexing : Different message bearing signal in different frequency range.
- Modulation makes it possible for the physical size of Tx. or Rx. Antenna to assume a practical value : Antenna size directly depends on wavelength or inversely to frequency. A higher frequency range gives lower antenna size.
- Modulation is used to shift the spectral content of a message signal so that it lies inside the operating frequency band of a communication channel :Converts wideband signal to a relative narrowband signal which solves many implementation issues. For shifting signal to a common frequency range to use same processing blocks.
- Modulation provides a mechanism for putting the information content of a message signal into a form that may be less volunerable to noise or interference

References: GATE Important Books - Communications

- Communication Systems by Simon Haykin, Wiley.
- Principle of Communication System by Taub ,Schilling & Saha, TMH.
- Modern digital and Analog Communications system by BP Lathi, Ding and Gupta, Oxford.
- Electronic Communication Systems by Kennedy and Davis (Noise chapter and Receivers), TMH.
- Signals and Systems by T K Rawat, Oxford
- Signals and Systems by Simon Haykin and V Veen, Wiley

THANK YOU

THANK YOU