# **POWER SYSTEM STABILITY**

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## **Outline of Presentation**

Introduction

✤Power angle curve

Dynamics Of Synchronous Machine

Swing equation

Analysis of steady state stability

✤Equal Area Criterion

Methods of improving stability

Previous years GATE Questions

- A large power system consists of a number of synchronous machines (or equipments or components) operating in synchronism.
- When the system is subjected to some form of disturbance, there is a tendency for the system to develop forces to bring it to a normal or stable condition
- The term stability refers to stable operation of the synchronous machines connected to a power system when they are subjected to sudden disturbances.
- Depending on the nature and magnitude of disturbances the stability studies can be classified in to the following types
  - 1. Steady state stability
  - 2. Transient stability



**Steady state stability** : It is defined as the ability of a power system to remain stable (i.e., without losing synchronism) for small disturbances (gradual changes in load).

- Static stability refers to inherent stability that prevails without the aid of automatic control devices
- Dynamic stability refers to artificial stability given to an inherently unstable system by automatic control devices. It is concerned with small disturbances lasting for 10 to 30 sec.

**The transient stability** is defined as the ability of a power system to remain stable for large disturbances. (such as sudden change in loads, loss of generations, excitations, transmission facilities, switching operations and faults).

#### Power Angle Curve

 $S=VI^*=P_e+jQ_e$ 

\* The graphical representation of power  $P_{\rm e}$  and the load angle  $\delta$  is called the power-angle curve.

.....(1)

#### Case (i): Generator loaded at its terminals

The complex power output of generator is

• 
$$P_e$$
=Real Power of (S)=  $\frac{|E||V|}{X_s} \sin \delta$ .....(4)

Thus, the real power output depends on and power angle  $\delta$ .

Power Angle Curve (contd...)



- $\clubsuit~$  As  $\delta$  is increased beyond 90°  $, \mathsf{P}_{e}$  decreases .
- At δ=180<sup>°</sup>,  $P_e$  becomes zero.
- ✤ Beyond δ=180<sup>°</sup>, P<sub>e</sub> becomes negative which implies that the power flow direction is reversed and the power is supplied from the infinite bus to the generator.
- The positive value of δ (E leading V) applies to generator action and negative value for δ (E lagging V) applies to the motor action.

### Power Angle Curve (contd...)

♦ The max steady-state power transfer occurs when  $\delta$ =90;

$$P\max = \frac{|E||V|}{X}....(5)$$

The value of P<sub>e,max</sub> is called the pull-out or steady-state stability limit.

- In actual practice  $\delta$  is kept round 30<sup>o</sup>
- \* When the power angle δ increases by a small amount  $\Delta\delta$ . The increase in synchronous power output is given by

Where P<sub>r</sub> synchronizing power coefficient

✤ In terms of ABCD parameters, power angle equation can be written as

$$P_{\max} = \frac{EV}{B} - \frac{AV^2}{B}\sin(\beta - \alpha)....(7)$$

Power Angle Curve (contd...)

Case (ii): Generator connected to Infinite bus.



The maximum steady state power transfer  $P_{max}$  occurs when  $\delta = 90^{\circ}$  and equals to  $\frac{|E||V|}{X} => Pmax = \frac{|E||V|}{X}$ .....(9)

**Transfer reactance(x):** The total reactance X between two voltage sources V and E is called the transfer reactance. The maximum power limit is inversely proportional to the transfer reactance.

#### Case (iii) : Power transfer through Impedance

- In all electrical machines and transmission lines, the resistance is negligible as compared to inductive reactance.
- ✤ Active power received by infinite bus is given by



♦ For  $P_{e,max}$  to be maximum (i.e. for max. power transfer), the above equation is to be differentiated w.r.t ' X'

$$\Rightarrow X = \sqrt{3}R....(11)$$

The practical application of (11) is limited. It that if X=0, power transferred is zero. Thus a finite value of reactance is necessary for power transfer.

# Methods of Improving Steady State Stability Limit

- The stability limit is the max. power that can be transferred in a network between source and load without loss of synchronism.
  - The steady state stability limit is the max. power that can be transferred without the system becoming unstable, when the load is increased gradually, under steady state condition.
  - Transient stability limit is the max. power that can be transferred with out the system becoming unstable when a sudden or large disturbance occurs.
- ✤ The transient stability is lower than the steady-state stability.

## Methods of Improving Steady State Stability Limit

✤ The steady state limit is given by SSSL=  $P_{\text{max}} = \frac{|E||V|}{X}$ where X is Transfer reactance.

- ✤ Methods to improve SSSL are
- 1. Operating the system at higher Voltages
- 2. Reducing the nett reactance of the system by
  - Parallel lines
  - Mid point compensation
  - Series capacitors
  - Double circuit
  - Bundle conductors

#### Dynamics of synchronous machine

- The kinetic energy of the rotor of the synchronous machine in terms of electrical angle is  $KE = \frac{1}{2}M\omega_s$ .....(12)
  Where,
  (P)
  - Where,  $\omega_s = \left(\frac{P}{2}\right) \omega_{sm} = rotor \, speed \, in \, rad(elec) / \sec$  $M = J \times \left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6} \, Moment \, of \, inertia \, in \, MJ. \sec/elec. rad$

✤ Inertia constant H can be defined through

Where *G* = machine rating in *MVA* 

H = inertia constant in MJ/MVA

Solving further,

$$\bigstar H = \frac{KE \text{ stored in rotor}}{MVA \text{ rating of aternator}}; \qquad M = \frac{H}{180f} \sec^2/\text{elec} - \text{rad}.....(13)$$

#### **Swing Equation**

The behavior of a synchronous machine during transients is described as swing equation.

Let,

- J = Total moment of inertia of the rotor mass in kg m<sup>2</sup>
- $\theta_{m}$ = Angular displacement of the rotor with respect to a stationary axis in mechanical radians.
- t = Time, in seconds.
- $T_m$ = The mechanical (or) shaft torques in N-m.
- $T_e$  = Net electrical (or) electromagnetic torque in N m.
- $T_a$  = Net accelerating torque in N m.
- Under steady state conditions,

 $T_e = T_m$  , N = constant

Tm Te GENERATOR  $\clubsuit$  When difference exists between  $F_m$  and  $F_e$  ,

 $T_a = T_m - T_e$  .....(14)

✤ By Newton's second law of motion,

✤ Further solving we get,

$$M\frac{d^2\theta}{dt^2} = \mathbf{P}_{\rm m} - \mathbf{P}_{\rm e} \dots (16)$$

\* The angular displacement  $\theta_e$  and  $\delta_m$  are related to synchronous



The above equation is called Swing Equation

#### Multi machine system

 $\clubsuit$  In a multi machine system a common system base must be chosen

Let  $G_{mach}$ = machine MVA rating (base)

 $G_{system}$ = system MVA base

#### Machines swinging coherently

Let the machines be  $M_1$  and  $M_2$ ,

\* Since the machine rotors swing together (coherently), =  $\delta_2 = \delta$ 

$$P_{m}=P_{m1}+P_{m2}$$
  
 $P_{e}=P_{e1}+P_{e2}$   
 $H_{eq}=H_{1}+H_{2}$ .....(20)

# Swing equation (contd...)

• The two machines swing coherently are thus reduced to a single machine as  $H_{1}$  and  $H_{2}$  and  $H_{2}$  and  $H_{2}$  and  $H_{2}$  and  $H_{3}$  and

- The above results are easily extendable to any no. of machines swinging coherently.
- Swing Curve: The graph between load angle and time is called Swing Curve.
- $\clubsuit$  If  $\delta$  increases continuously with time the system is unstable.

if  $\delta$  starts decreasing after reaching a maximum value it is said that the system will remain stable.



## Analysis Of Steady State Stability

- It is determined based on SSSL with power input, P<sub>m</sub> remaining same.
- Let us assume that the electrical power output increases by a small amount  $\Delta P$ .



- ♦ Now the torque angle is changed by a small amount Δδ.
  Therefore the new value of torque angle is (δ<sub>0</sub>+Δδ)
- \* The electrical power output for this new torque angle  $(\delta_0 + \Delta \delta)$  is given by  $P_e^1 = P_{e0} + \Delta P = P_{max} \sin(\delta_0 + \Delta \delta) \dots (22)$
- Since Δδ is small  $P_{e0} + \Delta P = P_{max} \sin \delta_0 + P_{max} \Delta \delta \cos \delta_0$ .....(23)
- $\Delta P = (P_{max} \cos \delta_0).....(24)$

#### Analysis Of Steady State Stability(Contd...)

For a torque angle of  $\delta = \delta_0 + \Delta \delta$ ,

$$M \frac{d^2}{dt} \left( \delta_0 + \Delta \delta \right) = P_m - (P_{e0} + \Delta P)$$

Since  $\delta_0$  is constant and  $P_m = P_{e0'}$ 

$$M \, \frac{d^2 \Delta \delta}{dt^2} = -\Delta P$$

Further solving, we obtain the following differential equation

$$M \frac{d^2}{dt} (\Delta \delta) + P_{\max} \cos \delta_0 \Delta \delta = 0$$
  
Let  $M \frac{d}{dt} (\Delta \delta) = x$ ,  $P_{\max} \cos \delta_0 = c$   
 $\Rightarrow x = \pm \sqrt{\frac{-P_{\max} \cos \delta_0}{M}} = \pm \sqrt{\frac{-\left(\frac{\partial P_e}{\partial \delta}\right)_0}{M}}$ .....(26)

## Analysis Of Steady State Stability(Contd...)

- **Case (i)** : When C is +ve( i.e.  $\left(\frac{\partial P_e}{\partial \delta}\right)_0 > 0$  or  $P_{\max} \cos \delta_0 > 0$ )
- Roots are purely imaginary and conjugate.
- \* The system behavior is oscillatory about  $\delta_0$ .

**Case (ii)** : When C is -ve (i.e  $\left(\frac{\partial P_e}{\partial \delta}\right)_0 < 0$  or  $P_{\text{max}} \cos \delta_0 < 0$ )

- The roots are real and equal in magnitude.
- ✤ One of the root is +ve and other is -ve.
- Due to the +ve root the torque angle increases without bound.
- When there is a small increment in power and machine will loose synchronism. Hence the machine becomes unstable for small changes in power provided.

# Analysis Of Steady State Stability(Contd...)

✤  $\left(\frac{\partial P_e}{\partial \delta}\right)_0$  is known as synchronizing coefficient. This is also called stiffness of synchronous machine.

#### Assumptions

- Generators are represented by constant impedances in series with no load voltages.
- The mechanical power input is constant.
- Damping is negligible.
- Load angle variations are small.
- Speed variations are negligible.

Transient stability limit is the maximum power that can be transferred without the system becoming unstable when a sudden or large disturbance occurs.

#### Assumptions:

- In transmissions line & synchronous machine resistance is neglected.
- Damping term contributed by synchronous machine damper winding is neglected.
- Rotor speed is assumed to be synchronous.
- Mechanical power input to machine remains constant.
- Voltage behind transient reactance is assumed remains constant.
- Loads are modelled as constant admittances.

The transient stability can be analysed by following methods

- i) Equal Area criterion.
- ii) Point by point method
- iii) Runga-Kutta method

#### Equal area criterion

The stability of a single machine connected to an infinite bus can be studied by the use of equal area criterion.

- ★ If P<sub>e1</sub>=P<sub>e0</sub>+ΔP then the accelerating power P<sub>a</sub> decreases from ΔP (when δ =δ<sub>0</sub>) to zero (when δ =δ<sub>1</sub>).
- During the time taken by the load angle to increase from  $\delta_0$  to  $\delta_1$ , the rotor absorbs KE. This KE equals to the shaded area  $A_1$ .

#### **Equal Area criterion**

- At point 'b' the P<sub>a</sub> = 0, but rotor acquires
   speed slightly greater than the synchronous
   speed, δ continues to increase beyond δ<sub>1</sub>.
- As  $\delta > \delta_1$ , p<sub>a</sub> becomes negative causing the rotor to retard.
- \* The rotor swing continues till the load angle is  $\delta_2$  and the rotor attains a speed equal to

synchronous speed.

- The load angle δ<sub>2</sub> can be obtained from the condition that the KE gained by rotor during its swing from δ<sub>0</sub> to δ<sub>1</sub> must equal to KE returned as it swing from δ<sub>1</sub> to δ<sub>2</sub>.
- This leads to conclusion that area  $A_1$  must be equal to shaded area  $A_2$ . This is referred as equal area criterion.





#### Mathematical way of expressing Equal area Criterion

• Multiplying eq (18) by  $2\left(\frac{d\delta}{dt}\right)$  and integrating, we get.

✤ For a stable system (i.e. the load angle will have minimum value when  $\frac{d\delta}{dt} = 0$ )
i.e.  $\int_{\delta_0}^{\delta} P_a d\delta = 0$ 

- This refers to zero<sup> $\delta_0$ </sup> area, which is possible only where  $p_a$  has both accelerating and decelerating power that is for,  $p_m > p_e$  and for the other part  $p_e > p_m$
- ✤ For the generator action  $P_m > P_e$  for positive area  $A_1$  and  $P_e > P_m$  for negative area  $A_2$  for stable operation.

## Sudden change in Mechanical Input

- Let the mechanical input to the generator rotor be suddenly increased to P<sub>m1</sub>.
- ✤ Since P<sub>m1</sub>>P<sub>e</sub> the generator will have

 $P_a = P_{m1} - P_e$ 

• In this new steady state  $P_{m1} = P_{e1}$ 

 $\therefore P_{m1} = P_{e1} = P_{max} sin\delta_1$ 

• The areas  $A_1$  and  $A_2$  can be evaluated as

$$A_{1} = \int_{\delta_{0}}^{\delta_{1}} (P_{m1} - P_{e}) d\delta$$
  

$$A_{2} = \int_{\delta_{2}}^{\delta_{1}} (P_{e} - P_{m1}) d\delta$$
(28)



# Sudden change in Mechanical Input (Contd...)

- As  $p_m$  increases, a limiting condition is finally reached at a point where the area  $A_1=A_2$
- $\clubsuit$  The corresponding  $\delta_1$  can be  $\delta_{1max}$  and  $\delta_2$  be  $\delta_{2max}.$
- Here  $\delta_{2max} = \pi \delta_{1max}$

Since 
$$\delta_{1,\max} = \sin^{-1} \left( \frac{P_{m1,\max}}{P_{\max}} \right)$$
  
 $\implies \delta_{2\max} = \prod -\sin^{-1} \left( \frac{P_{m1,\max}}{P_{\max}} \right)$ 

#### Critical clearing angle & Critical clearing time

Critical clearing angle ( $\delta_{cc}$ ): It is the maximum allowable change in the power angle δ, before clearing the fault, without loss of synchronism.

$$\delta = \delta_0 + \frac{P_a}{2M}t^2 \qquad \delta_{cc} = \cos^{-1} \left[\cos \delta_{\max} + \frac{P_m}{P_{\max}}(\delta_{\max} - \delta_0)\right]....(29)$$

- Critical clearing time(t<sub>cc</sub>): It can be defined as the maximum time delay that can be allowed to clear the fault without loss of synchronism.  $t_{cc} = \sqrt{\frac{2H(\delta_{cc} \delta_0)}{\prod fP}}.....(30)$
- If the actual clearing angle is greater than the Critical clearing angle, the system is unstable, otherwise it is stable.

## Sudden Loss of one of the parallel line

• When both of the lines are operating the power is given by  $P_{e1} = \frac{EV}{X_{d_1} + \frac{X_1X_2}{X_1 + X_2}} \sin \delta = P_{\max I} \sin \delta$ .....(31)



When one of the lines is switched out the transfer reactance increases and the power transfer is given by

$$P_{e^2} = \frac{EV}{X_{d1} + X_1} \sin \delta = P_{\max II} \sin \delta....(32)$$

$$\Rightarrow P_{\max I} > P_{\max II}$$

 The maximum value which can attain without loss of system stability is  $\delta_{\rm m}$  and equals to

 $(\pi-\delta_1)$  radians.



## Fault and Subsequent circuit isolation

- When fault develops at any point F on line2 it is subsequently cleared by cleared by opening the CBS at both the ends of faulted line.
- ✤ 3 power angle curves are involved,
  - first for the pre fault system,
  - second for the system during fault and
  - third for the system after the fault line
- ♦ If actual clearing  $\delta_c < \delta_{cc}$  system is stable,
- If  $\delta_c > \delta_{cc}$  the system is unstable.



## Fault and Subsequent circuit isolation(Contd....)

An expression for  $\delta_{cc}$  can be derived as

Where,

$$\delta_{\max} = \prod -\sin^{-1} \left( \frac{P_m}{P_{\max III}} \right) \qquad \delta_0 = \sin^{-1} \left( \frac{P_m}{P_{\max I}} \right)$$



## Fault, Circuit isolation and reclosing

- The transmission lines are provided with automatic quick reclosing circuit breakers, because most of the faults are transient in nature
- When fault occurs, operation shifts to curve for faulted condition.
- \* When the load angle is  $\delta_{c}$ , the faulted line is isolated and the operation shifts to the post fault curve.
- \* When the load angle is  $\delta_0$  the circuit breaker reclose and operation shifts to pre-fault curve.
- For stable operation the accelerating area  $A_1$  = decelerating area  $A_2$ .
- The maximum angle to which rotor swings is δ<sub>2</sub> and is less than δ<sub>m</sub> (i.e the maximum permissible rotor swing if stability is to be maintained).

#### Solution of swing equation by point by point method

- Point by point (or step by step) method is the most feasible and widely used way of solving the swing equations.
- The main assumption for solving the swing equation by point by point method is "the accelerating power is constant during time interval".
- Integrating swing equation twice, w.r.t. time't',
- ★ After 1st integration,  $\frac{d\delta}{dt} = \omega = \omega_0 + \frac{P_a}{M}$ ......(34)
  ★ After 2 nd integration,  $\delta = \delta_0 + \omega_0 t + \frac{P_a}{2M}$ ......(35)
- ✤ Dividing the total time 't' into 'n' equal intervals.

$$\omega_{n} = \omega_{n-1} + \frac{\Delta t}{M} P a_{n-1}.....(36) \qquad \delta_{n} = \delta_{n-1} + \Delta t . \omega_{n-1} + \frac{(\Delta t)^{2}}{2M} P a_{n-1}....(37)$$

#### Solution of swing equation by point by point method(Contd....)

✤ The increments of speed and angular displacement during the n<sup>th</sup> interval

$$\Delta \omega_n = \omega_n - \omega_{n-1} = \frac{\Delta t}{M} P a_{n-1} \dots (38)$$
$$\Delta \delta_n = \Delta \delta_{n-1} + \frac{(\Delta t)^2}{2M} (P a_{n-1} + P a_{n-2}) \dots (39)$$

# Methods of improving stability

- By increasing inertia constant(M)
- Increasing system voltage
- Reduction of transfer reactance
  - Use of double circuit lines
  - Use of Bundle conductors
  - Series compensation of the lines
- ✤ Fast switching
- Turbine fast valving (or) By-pass valving
- Single –pole switching
- ✤ Load shedding
- HVDC links
- Breaking resistors

Methods of improving stability (Contd.....)

- Short circuit current limiters
- Full load rejection technique

Q. No.1) Steady state stability of a power system is the ability of the system to (GATE- 99)

- A. Maintain Voltage at the rated level.
- B. Maintain frequency exactly at 50Hz.
- C. Maintain spinning reverse margin at all times.
- D. Maintain Synchronism between machines and on external tie lines.
- Sol) A

Q. No.2) The transient stability of the power system can be effectively improved by (GATE-93)

- A) Excitation control
- B) Phase shifting transformer
- C) Single pole switching of circuit breakers
- D) Increasing the turbine valve opening

Sol. C

Q. No.3) The angle  $\delta$  in the swing equation of a synchronous generator is (GATE-13)

- A) Angle between stator voltage and current.
- B) Angular displacement of stator with respect to rotor.
- C) The angular displacement mmf with respect to a synchronously rotating axis.
- D) Angular displacement of an axis field to the rotor with respect to a synchronously rotating axis.  $\gamma$ , we represent the rotor field to the rotor with respect to a synchronously rotating axis.

Sol. D



Q. No.4) In the single machine infinite bus system shown below, the generator is delivering real power of 0.8pu at 0.8 p.f lagging to the infinite bus. The power angle of the generator in degrees is\_\_\_\_(GATE-2019)



Q. No.5) A transmission line has a series reactance of 0.2 pu. Reactive power compensation is applied at the midpoint of the line and it is controlled such that the midpoint voltage of the transmission line is always maintained at 0.98pu. If voltage at both ends of the line are maintained at1.0pu, then the steady state power transfer limit of the transmission line is (GATE-02)

Sol: 
$$|V_1| = |V_2| = 1$$
 pu

$$|E| = 0.98 \text{ pu}$$

$$X = 0.2 \text{ pu}$$
Substituting in  $P \max = \frac{|E||V|}{X/2}$ 

$$= \frac{0.98 \times 1}{0.1} = 9.8 \text{ pu}$$

Q. No.6) A 500MW, 21 KV, 50Hz, 3-phase, 2-pole synchronous generator having rated p.f.= 0.9, has a moment of inertia 27.5×  $10^3 \text{ kg } m^2$ . The inertia constant H will be (GATE-09)

Sol.  $H = \frac{KE \ stored \ in \ rotor}{MVA \ rating \ of \ aternator};$ 

Where KE = 
$$\frac{I \omega^2}{2}$$
 and  $\omega = \frac{2\pi N}{60}$   
N= $\frac{120 \times 50}{2}$  = 3000 rpm  
 $\omega = \frac{2\pi \times 3000}{60}$  = 314.15 rad/sec  
KE =  $\frac{27.5 \times 10^3 \times 314.15^2}{2}$ ; H= $\frac{1357}{\frac{500}{0.9}}$  = 2.44 MJ/MVA

Q. No.7) A 50Hz, 4 pole, 500MVA,22Kv, turbo-generator is delivering rated megavolt-amperes at 0.8 power factor. Sudeenly fault occurs reducing its power output by 40%. Neglect losses and assume constant power input to the shaft. The accelerating torque in the generator in MNm at the time of fault will be (GATE-04)

Sol.

Before fault;  $P_m = P_{e1} = 500 \text{MVA x } 0.8 = 400 \text{MW}$ During Fault;  $P_{e2} = 0.6 \text{ x } 400 \text{MVA} = 240 \text{MW}$  $P_a = P_{e1} - P_{e2} = 400 - 240 = 160 \text{ MW}; \text{ N} = 1500 \text{rpm};$  $\omega = \frac{2\pi N}{60} = 157 \text{ rad/sec};$  $T_a = \frac{P_a}{60} = 1.018 \text{MNm}.$  Q. No.8) A 50Hz generating unit has H-constant of 2MJ/MVA. The machine is initially operating in steady state at synchronous speed, producing 1pu of real power. The initial value of rotor angle is 5°, When a bolted 3- $\Phi$  to ground short circuit fault occurs at the terminals of the generator. Assuming the input mechanical power to remain at 1 pu, the value of  $\delta_{,}$  0.02 seconds after fault is\_\_\_\_ (GATE-15)

Sol) 5.7 to 6.1

$$\delta_0 = 5, P_a = 1pu, t=0.02s$$
  
$$\delta = \delta_0 + \frac{P_a}{2M}t^2; \quad M = \frac{H}{180f}$$
  
$$= 5 + [\frac{1}{2} \times \frac{1}{\frac{2}{180 \times 50}} \times 0.02^2] = 5.9^{\circ}$$

#### Previous years GATE Questions

Q. No.9) A lossless single machine infinite bus power system is shown in the fig. The synchronous generator transfers 1.0pu of power to infinite bus. The critical clearing time of circuit braker is 0.28 s. If another identical synchronous generator is connected in parallel to the existing generator and each generator is scheduled to supply 0.5pu of power, the critical clearing time of the circuit breaker will be (GATE-09)

Sol.

$$M_{(p.u)}\frac{d^2\delta}{dt^2} = Pa$$

1.0∠δ pu .0∠0 pu 1.0∠0 pu 1.0 pu

 $P_{e}$ = 0 during fault (Electrical power delivered )

$$\frac{d^2\delta}{dt^2} = Pa/M; \qquad \delta = \frac{P_a}{2M}t^2 + A;$$

For initial conditions, t=0;  $\delta = \delta_0$ 

$$\delta_0 = \frac{P_a}{2M}(0) + A$$

Critical clearing time

$$t_{cc} = \sqrt{\frac{2H(\delta_{cc} - \delta_0)}{\prod f P_m}}$$

Initially when one machine is delivering 1.0 pu, electrical o/p

$$P_s = P_e = 1.0 \text{ pu}, M_1 = M$$

When two identical generators are delivering a total electrical o/p of 1.0 pu

$$P_{s \text{ total}} = P_{e \text{ total}} = 1.0 \text{ pu}, M_2 = 2M$$

$$t_{c1} = 0.28 = \sqrt{\frac{(\delta_{c1} - \delta_0)2M_1}{P_s}}$$

Previous years GATE Questions

$$t_{c2} = \sqrt{\frac{(\delta_{c2} - \delta_0)2M_2}{P_{s \text{ total}}}}$$

$$=\sqrt{\frac{(\delta_{c2}-\delta_0)4M}{P_s}}$$

Therefore critical clearing time of breaker will increase beyond 0.28s

Q. No.10) The figure shows single line diagram of power system with a double circuit transmission line. The expression for electrical power is  $1.5 \sin \delta$ , where  $\delta$  is the rotor angle. The system is operating at the stable equilibrium point with mechanical power 1 pu. If one of the transmission line circuits is removed, the maximum value of  $\delta$ , as the rotor swings is 1.221 radian. If the expression for electrical power with one transmission line circuit removed is  $P_{max}\sin\delta$ , the value of  $P_{max}$  in pu is \_\_\_(GATE-17)

Sol) 1.22

Continued in the next page.

#### Previous years GATE Questions

Q. No.10)  $\delta_2 = \delta_m = 70$  $P_s = P_{e1} = P_{m1} \sin \delta_0$  $\delta_0 = sin^{-1} \left[ \frac{P_s}{P_{m1}} \right] = sin^{-1} \left[ \frac{1.0}{1.5} \right]$  $\delta_0 = 0.728 \text{ rad}$  $=1.2221 \text{ x} \frac{180}{3.14} = 70$  $A_1 + A_2 = 0$ area under graph using integration gives  $\int_{0}^{\delta_{1}} (P_{s} - P_{e2}) d\delta + \int_{0}^{\delta_{2}} (P_{s} - P_{e2}) d\delta = 0$ 



Q. No.10)

 $P_s\,\delta_1\text{-}P_s\,\delta_0\text{+}\,P_{m2}\,\text{cos}\delta_0\text{+}P_s$ 

 $P_{s}(\delta_{1}-\delta_{0}) + P_{m2}(\cos\delta_{2}-\cos\delta_{0})=0$ 

 $1.0(1.221-0.728) + P_{m2}(\cos 70 - \cos 41.75)=0$ 

 $0.493+ P_{m2}(0.342-0.746)=0$ 

$$P_{m2} = \frac{0.493}{0.404}$$

$$P_{m2}$$
 = 1.22 pu

Q. No.11) A cylindrical rotor generator delivers 0.5pu power in the steady state to an infinite bus through a transmission line of reactance 0.5pu. The generator no-load voltage is 1.5pu and the infinite bus voltage is1 pu. The inertia constant of the generator is 5MW- s/MVA and the generator reactance is 1pu. The critical clearing angle in degrees, for a 3-phase dead short circuit fault at the generator terminal is

Sol:

$$P_{s} = P_{e1} = 0.5$$
  
Before fault  $P_{m1} = \frac{EV}{X}$ 
$$= \frac{1.5 \times 1.0}{1.5} = 1.0$$

Q. No.11) During Fault  $P_{m2}=0$ After Fault P<sub>m3</sub>=1.0  $\delta_0 = sin^{-1} \left[ \frac{P_s}{P_{m1}} \right]$  $= sin^{-1} \left[ \frac{0.5}{1.0} \right] = 30^{\circ}$  $\delta_0$ (radians)= $\frac{30 \times \pi}{180}$  = 0.52 rad  $\delta_{\text{max}} = 180 - sin^{-1} \left[ \frac{P_s}{P_{\text{max}}} \right]$  $= 180 - sin^{-1} \left[ \frac{0.5}{1} \right] = 150^{\circ}$ 

#### Previous years GATE Questions

Q. No.11)

$$\delta_0$$
(radians)= $\frac{150 \times \pi}{180}$  = 2.618 rad

Critical clearing angle 
$$\delta_{c} = Cos^{-1} \left[ \frac{P_s(\delta_{\max} - \delta_0) + P_{m3} \cos \delta_{\max}}{P_{m3}} \right]$$

$$= Cos^{-1} \left[ \frac{0.5(2.618 - 0.52) + 1.0 \cos 150}{1.0} \right]$$

= 79.45 degrees

Q.No.12) A power station consists of two synchronous generators A & B of ratings 250 MVA and 500 MVA with inertia constant 1.6pu and 1.0pu respectively on their own base MVA ratings. The equivalent pu inertia constant for the system on 100 MVA common base is (GATE-98)

Solution:

Inertia constant H  $\alpha \frac{1}{MVA rating(s)}$ 

$$H_{Anew} = H_{Aold} \times \frac{S_{old}}{S_{new}}$$
$$= 1.6 \times \frac{250}{100}$$
$$= 4.0 \text{ pu}$$

#### Previous years GATE Questions

Q.No.12

$$H_{Bnew} = H_{Bold} \times \frac{S_{old}}{S_{new}}$$
$$= 1.0 \times \frac{500}{100}$$
$$= 5.0 \text{ pu}$$
$$H_{eq} = H_{Anew} + H_{Bnew}$$
$$= 4.0 + 5.0$$
$$= 9.0 \text{ pu}$$

### Previous years GATE Questions



Block diagram of two-area load frequency control