
ECONOMIC LOAD DISPATCH

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Outline of Presentation

- ❖ Introduction
- ❖ System constraints
- ❖ Characteristics
- ❖ Methods of solving ELD
- ❖ Loss coefficients
- ❖ Previous years GATE Questions

Introduction

- ❖ The main aim in the economy of operation problem is to minimize the total cost of generating real power at various stations while satisfying the loads and the losses in the transmission links.
- ❖ Economy of operation is naturally predominant in determining allocation of generation to each station for various system load levels.
- ❖ The first problem in power system is called the unit commitment (UC) problem and the second is called the load scheduling (LS) problem.

Introduction (contd...)

- ❖ Economic dispatch is the on line economic dispatch where in it is required to distribute the load among the generating units actually paralld with the system in such manner as to minimize the total cost of supplying the minute – to – minute requirements of the system.
- ❖ Economic load dispatch problem is really the solution of a large number of load flow problems and choosing the one which is optimal in the sense that it needs minimum cost of generation.

System constraints

❖ There are two types of constraints.

(1) Equality constraints

(2) In equality constraints.

1) Equality constraints:

The equality constraints are the basic load flow equations given by

$$P_p = \sum_{q=1}^n [e_p(e_q G_{pq} + f_q B_{pq}) + f_p(f_q G_{pq} - e_q B_{pq})]$$

$$Q_p = \sum_{q=1}^n [f_p(e_q G_{pq} + f_q B_{pq}) - (f_q G_{pq} - e_q B_{pq})]$$

$$P = 1, 2, \dots, n$$

System constraints(contd..)

2) Inequality constraints:

They are

- a) Generator constraints
- b) voltage constraints
- c) Transformer tap settings
- d) Transmission line constraints.
- e) Network security constraints.

a) Generator Constraints:

The kVA loading on a generator is given by $\sqrt{P_p^2 + Q_p^2}$ and this should not exceed a pre specified value C_p because of the temperature rise conditions that is $P_p^2 + Q_n^2 \leq C_p^2$.

System constraints(contd..)

b) Voltage Constraints:

It is essential that the voltage magnitudes and phase angles at various nodes should vary within certain limits. The voltage magnitude should vary within certain limits because otherwise most of the equipment connected to the system will not operate satisfactorily or additional use of voltage regulating device will make the system uneconomical. Thus

$$|V_{p_{min}}| \leq |V_p| \leq |V_{p_{max}}|$$

$$\delta_{p_{min}} \leq \delta \leq \delta_{p_{max}}$$

System constraints(contd..)

c) Transformer Tap Settings:

If an auto transformer is used the minimum tap settings could be 0 and the maximum 1

i.e. $0 \leq t \leq 1$

Similarly for a two winding transformer if tapings are provided on the secondary side

$0 \leq t \leq n$, where n is the ratio of transformation. Phase shift limits of the phase shifting transformer.

$$\theta_{p_{min}} \leq \theta \leq \theta_{p_{max}}$$

System constraints(contd..)

d) Transmission Line Constraints:

The flow of active and reactive power through the transmission line circuit is limited by the thermal capability of the circuit and is expressed as $C_p \leq C_{p_{max}}$, where $C_{p_{max}}$ is the maximum loading capacity of the pth length.

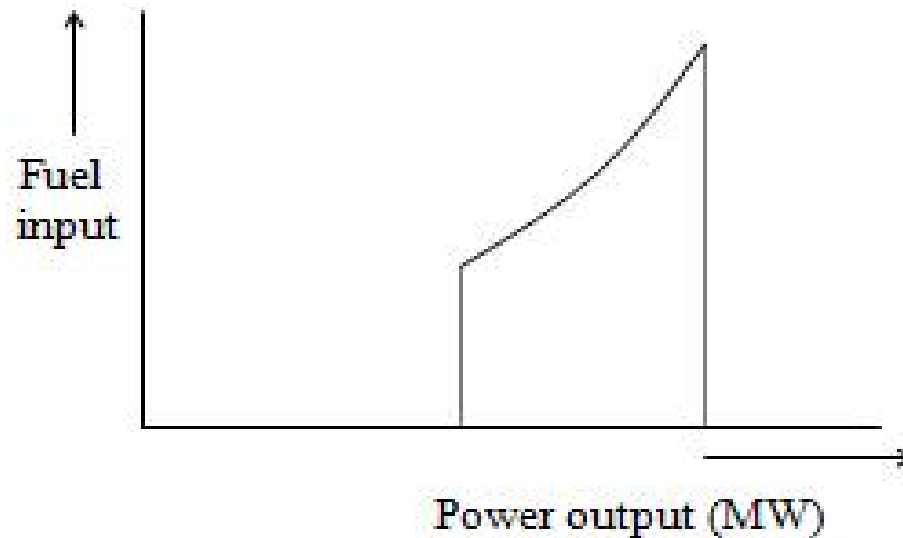
e) Network Security Constraints:

If initially a system is operating satisfactorily and there is an outage, may be scheduled or forced one, it is natural that some of the constraints of the system will be violated. The complexity of these constraints is increased when a large system is under study.

Characteristics

1) Input-output characteristics:

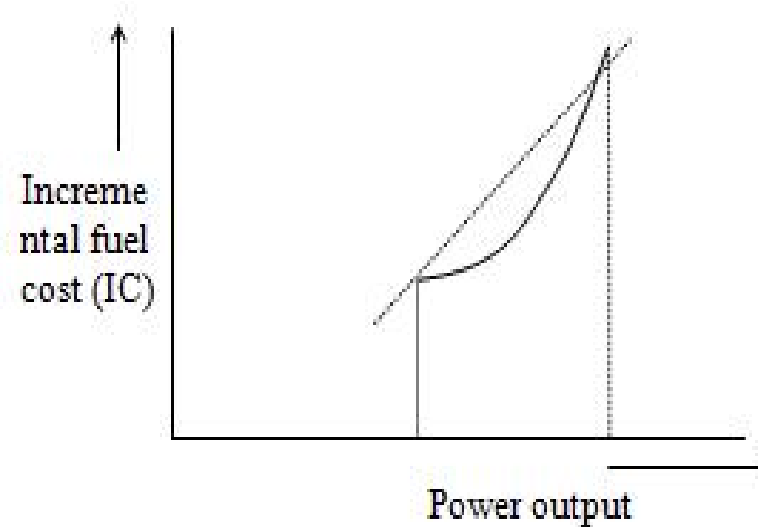
The input-output curve of a unit can be expressed in Million kilocalories per hour (or) directly in terms of rupees per hour versus output in megawatts.



Characteristics(contd..)

2) Incremental fuel cost:

It is a ratio equal to a small change in input to the corresponding small change in output.



Characteristics(contd..)

3)Incremental production cost:

Incremental production cost consists of the incremental fuel cost plus the incremental cost of labour, supplies, maintenance and water. It is difficult to express exactly these costs as a function of output and also they form generally a small fraction of the incremental cost of fuel, the incremental cost of production will wither to be considered equal to the incremental cost of fuel.

Methods of solving ELD

1) By neglecting losses:

$F_t \rightarrow$ is the total fuel input to the system

$F_n \rightarrow$ is the fuel input to nth unit

$P_d \rightarrow$ is the total load demand.

$P_n \rightarrow$ is the generation of nth unit.

The economic dispatch problem is defined as

$$\text{Min } F_T = \sum_{n=1}^n F_n$$

$$\text{Subject to } P_D \quad P_D = \sum_{n=1}^n P_n$$

By using a Lagrangian multiplier technique, we arrive at a solution where,

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \frac{dF_3}{dP_3} = \dots = \lambda \quad (1)$$

Here $\frac{dF_1}{dP_1}$ is the incremental cost of generation at plant 1 in unit currency/hr and so on.

Methods of solving ELD(contd..)

1)By considering losses:

$F_t \rightarrow$ is total fuel input to the system

$F_n \rightarrow$ is the fuel input to nth unit

$P_d \rightarrow$ is the total load demand

$P_n \rightarrow$ is the generation of nth unit

$P_l \rightarrow$ is the total system loss

The optimal load dispatch problem including transmission losses is defined as

$$\text{Min } F_T = \sum_{n=1}^n F_n$$

$$\text{Subject to } P_D + P_L = \sum_{n=1}^n P_n$$

Methods of solving ELD(contd..)

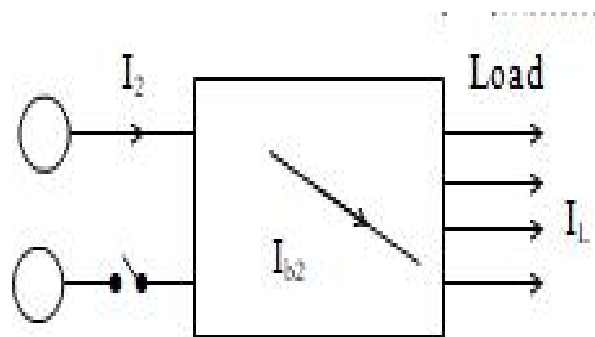
Solving this using the Lagrangian multiplier again, we arrive at,

$$P_n = \frac{1 - \frac{f_n}{\lambda} - \sum_{m \neq n} 2B_{mn}P_m}{\frac{F_{nn}}{\lambda} + B_{nn}}$$

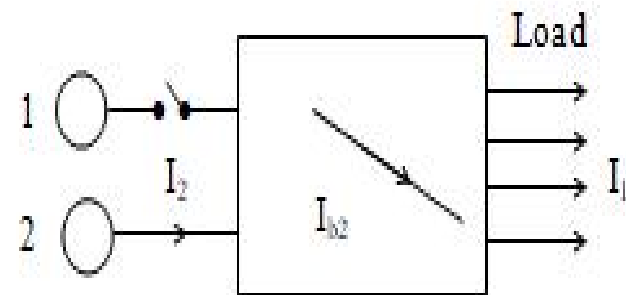
With a certain coordination equation written as,

$$F_{nn}P_n + f_n + \lambda \sum 2B_{mn}P_n = \lambda$$

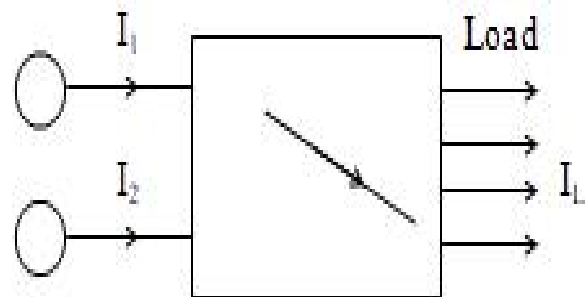
Loss coefficients



(a)



(b)



(c)

Loss coefficients

There are two simplifying assumptions for deriving the loss coefficients.

- (i) The ratio for the transmission lines is the same.
- (ii) The phase angle of all the load currents is the same.

By doing necessary calculations we obtain the basic loss equation as

$$P_1 = B_{11}p_1^2 + B_{22}p_2^2 + 2P_1P_2B_{12}$$

$$\text{Where } B_{11} = \sum \propto b_1^2 R_b / |v_2|^2 \cos^2 \phi_2$$

$$B_{22} = \sum \propto b_2^2 R_b / |v_2|^2 \cos^2 \phi_2$$

$$B_{12} = \sum \propto b_1 \propto b_2 \cos(\theta_1 - \theta_2) R_b / |v_1| |v_2| \cos \phi_m \cos \phi_n$$

Previous years GATE Questions

Q.No.1) An industrial consumer has a daily load pattern of 2000 KW ,0.8 lag for 12 hours and 1000KW UPF for 12 hours. The load factor is (GATE-99)

Sol)As we know

$$\text{Load factor} = \frac{\text{Actual number of units generated}}{\text{max load} \times \text{Total no. of hours}}$$

$$= \frac{(2200 \times 12) + (1000 \times 12)}{2000 \times 24}$$

$$= 0.75$$

Previous years GATE Questions

Q. No.2) The incremental cost characteristics of two generators delivering 200 MW are as follows (GATE-00)

$$\frac{dF_1}{dP_1} = 20 + 0.1P_1$$

$$\frac{dF_2}{dP_2} = 16 + 0.2P_2$$

For economic operation the generations P_1 and P_2 should be

Sol)

$$P_1 + P_2 = 200 \text{ ----- (a)}$$

By referring to eqn 1 we will get another eqn i.e.,

Previous years GATE Questions

$$P_2 = 20 + 0.5P_1 \text{ ----- (b)}$$

By solving eqn a and b we get

$$P_1 = 120 \text{ MW} \quad P_2 = 80 \text{ MW}$$

Q.No.3) Incremental fuel costs for a power plant consisting of three generating units are

$$IC_1 = 20 + 0.3P_1, IC_2 = 30 + 0.4P_2, IC_3 = 30 \quad \text{(GATE-03)}$$

Assume that all the 3 units are operating all the time. Minimum and maximum loads on each unit are 50 and 300 MW respectively. If the plant is operating on economic load dispatch to supply the total demand of 700 MW, the power generated by each unit is

Previous years GATE Questions

Sol) As the incremental fuel cost of gen 3 is independent of its generation, we can operate it at its maximum capacity of 300 MW and the remaining 400 MW will be shared by remaining two generators

$$P_1 + P_2 = 400 \text{-----(a)}$$

By referring to eqn 1

$$20 + 0.3P_1 = 30 + 4P_2 \text{-----(b)}$$

From eqns a and b

$$20 + 0.3P_1 = 30 + 4(400 - P_1)$$

By solving it we get

$$P_1 = 242.86 \text{ MW}$$

$$P_2 = 157.14 \text{ MW}$$

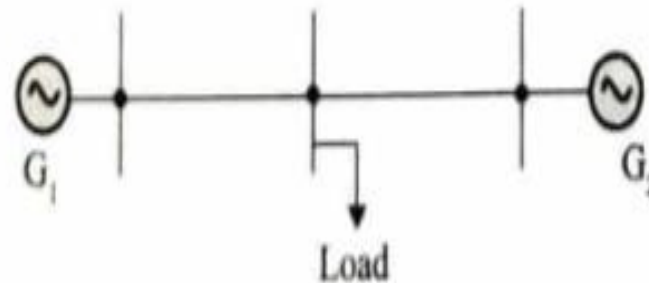
$$\text{And } P_3 = 300 \text{ MW}$$

Previous years GATE Questions

Q.N0.4) A load is at equidistant from the two thermal generating stations G1 and G2 as shown in fig. The fuel cost characteristics of the generating stations are given by

$$F_1 = a + bP_1 + cP_1^2 \text{ Rs/hour}$$

$$F_2 = a + bP_2 + 2cP_2^2 \text{ Rs/hour} \quad (\text{GATE-05})$$



Where P_1 and P_2 are the generations in MW of G1 and G2 respectively. For most economic generation to meet 300 MW of load P_1 and P_2 are

Previous years GATE Questions

Sol) By differentiating the input fuel cost equations we get

$$IC_1 = b + 2cP_1$$

$$IC_2 = b + 4cP_2$$

From eqn 1 for most economic operation

$$b + 2cP_1 = b + 4cP_2$$

$$\text{and } P_1 + P_2 = 300$$

By solving them

$$P_1 = 200 \text{ MW}$$

$$P_2 = 100 \text{ Mw}$$

Previous years GATE Questions

Q. No.5) A lossless power system has to serve a load of 250 MW. There are two generators in the system with cost curves C_1 and C_2 respectively defined as follows

$$C_1 = P_{G1} + 0.055 \times P_{G1}^2$$

$$C_2 = 3P_{G2} + 0.03 \times P_{G2}^2$$

Where P_{G1} and P_{G2} are the MW injections from generator G_1 and G_2 respectively. Thus the minimum cost dispatch will be

Sol) From data $P_{G1} + P_{G2} = 250$ ----- (a)

From eqn 1

$$1 + 0.11P_{G1} = 3 + 0.06P_{G2}$$
----- (b)

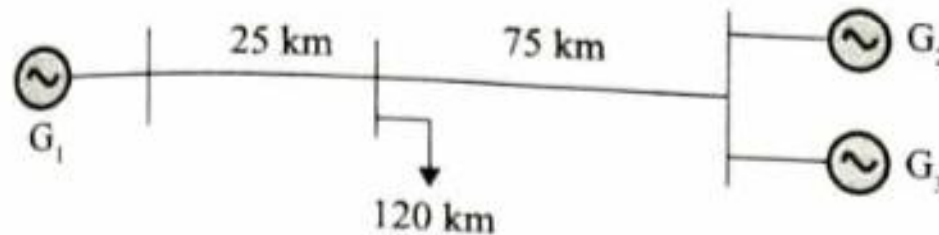
Solving eqn a and b we get

$$P_{G1} = 100 \text{ MW}$$

$$P_{G2} = 150 \text{ MW}$$

Previous years GATE Questions

Q.No.6) A load center of 120 MW deliver power from two power stations connected by 220 KV transmission lines of 25 Km and 75 Km as shown in fig below. The three generators are of 100 MW capacity each and have identical fuel cost characteristics. The minimum loss generation schedule for supplying the 120 MW load is



Previous years GATE Questions

- a) $P_1=80+\text{losses}$, $P_2=20$ MW, $P_3=20$ MW
- b) $P_1=60$ MW, $P_2=30$ MW+ losses, $P_3=20$ MW
- c) $P_1=40$ MW, $P_2=40$ MW, $P_3=40$ MW+ losses
- d) $P_1=30+\text{losses}$, $P_2=45$ MW, $P_3=45$ MW

Sol) Generator 1 is near to the load centre, where as Generator 2 and 3 are away from the load . To reduce losses , more power is contributed from generator 1

$$P_1=80+\text{losses}, P_2=20 \text{ MW}, P_3=20 \text{ MW}$$

Previous years GATE Questions

Q.No.7) The fuel cost functions of two power plants are

$$\text{Plant } P_1: C_1 = 0.05P_{g1}^2 + AP_{g1} + B$$

$$\text{Plant } P_2: C_2 = 0.01P_{g2}^2 + 3AP_{g2} + 2B$$

Where P_{g1} and P_{g2} are the generator powers of two plants and A and B are constants. If the two plants optimally share 1000 MW load at incremental fuel cost of 100 Rs/MWh, the ratio of load shared by the two plants is (GATE-154-S1)

Sol)

$$P_{g1} + P_{g2} = 1000 \text{ MW} \text{---(a)}$$

Refer eqn 1

$$0.1 P_{g1} + A = 100 \text{-----(b)}$$

$$0.2 P_{g2} + 3A = 100 \text{-----(c)}$$

Solve eqn 4, 5, 6 we get

$$P_{g1} = 800 \text{ MW}, P_{g2} = 200 \text{ MW} \quad P_{g1} : P_{g2} = 4:1$$

Previous years GATE Questions

Q.No.8)The incremental cost (in rupees / MWh) of operating two generating units are functions of their respective powers P_1 and P_2 in MW, and are given by

$$\frac{dC_1}{dP_1} = 0.2 P_1 + 50$$

$$\frac{dC_2}{dP_2} = 0.24 P_2 + 40$$

Where $20 \text{ MW} \leq P_1 \leq 150 \text{ MW}$

$20 \text{ MW} \leq P_2 \leq 150 \text{ MW}$

For a certain load demand, P_1 and P_2 have been chosen such that $\frac{dc_1}{dp_1} = 76 \text{ Rs/MWh}$ and $\frac{dc_2}{dp_2} = 68.8 \text{ Rs/MWh}$. If the generations are rescheduled to minimize the total cost then P_2 is

Previous years GATE Questions

Sol) By referring to eqn 1 and by solving it we get

$$P_1 = 80 \text{ MW } P_2 = 50 \text{ MW}$$

$$\text{Lambda syn} = 0.5(50) + 40 \\ = 65 \text{ Rs/MWh.}$$

Q.No.9) The fuel cost of generators G_1 and G_2 are $C_1(P_{G1}) = 10,000$ Rs/MWhr and $C_2(P_{G2}) = 12,500$ Rs/MWhr and the loss in the line is $P_{loss(pu)} = 0.5 * P_{G1(pu)}^2$ where the loss coefficient is specified in p.u. on a 100 MVA base. The most economic power generation schedule in MW is

Sol) For economic load dispatch,

$$L_1 \frac{dF_1}{dP_{G1}} = L_2 \frac{dF_2}{dP_{G2}}$$

Previous years GATE Questions

$$\left(\frac{1}{1 - \frac{dP_L}{dP_{G1}}} \right) \frac{dF_1}{dP_{G1}} = \left(\frac{1}{1 - \frac{dP_L}{dP_{G2}}} \right) \frac{dF_2}{dP_{G2}}$$

From the given data,

$$\frac{dP_L}{dP_{G1}} = 2(0.5)P_{G1} = P_{G1}$$

$$\frac{dP_L}{dP_{G2}} = 0$$

$$\left(\frac{1}{1 - P_{G1}} \right) 10,000 = \left(\frac{1}{1 - 0} \right) 12,500$$

Previous years GATE Questions

$$1 - P_{G1} = \frac{100}{125}$$

$$\Rightarrow P_{G1} = 0.2 \text{ p.u.}$$

$$\Rightarrow P_{G1} = 0.2 * 100 = 20 \text{ MW}$$

$$\text{But } P_L = 0.5 * P_{G1}^2$$

$$= 0.5(0.2^2)$$

$$= 0.02 \text{ p.u.}$$

$$P_L = 0.02 * 100 = 2 \text{ MW}$$

$$P_{G1} + P_{G2} = P_D + P_L$$

$$20 + P_{G2} = 40 + 2$$

$$\Rightarrow P_{G2} = 22 \text{ MW}$$

Thank You